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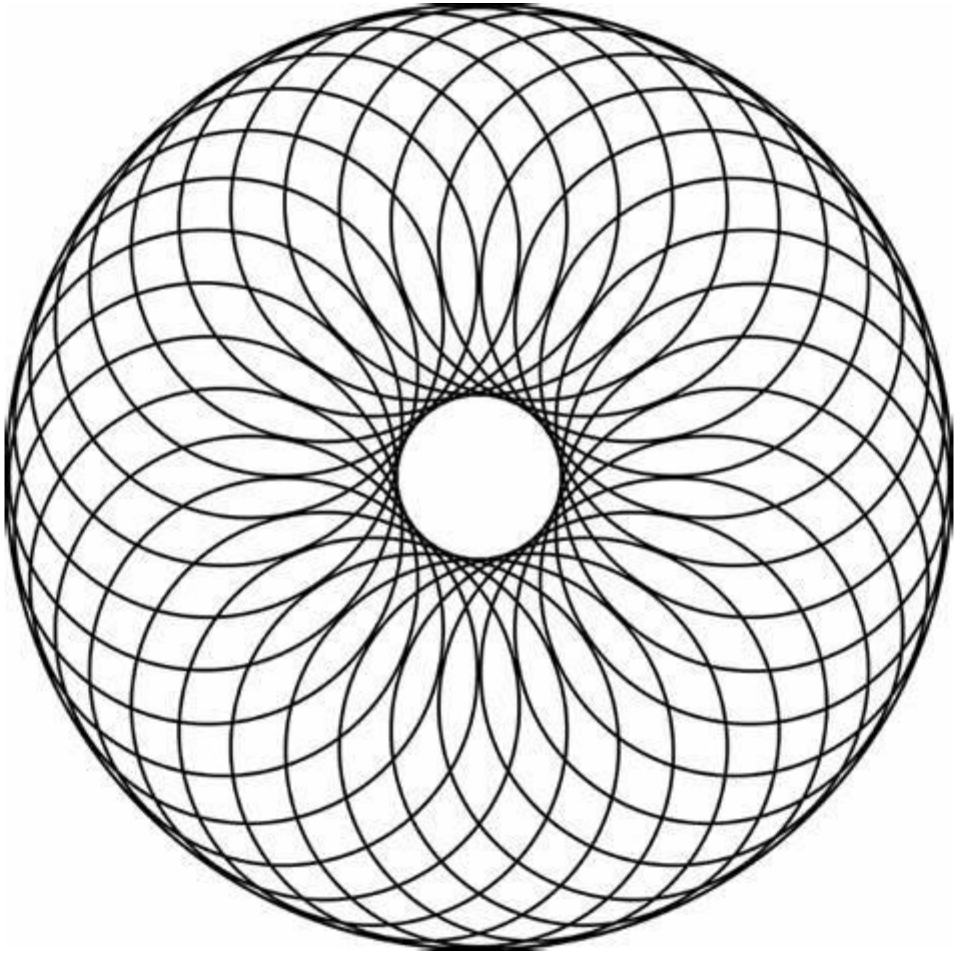
A LEVEL MATHS

YEAR 1 PURE



This book started off as homework sheets to encourage my students to do work at home straight after the lesson we had just done. It ended up turning into this, a book, one intended to help guide you through the A Level Maths course with questions of all difficulties.

The questions aren't C/D or A/A* questions as such, just the type of skill level who is working at that grade.

The questions have evolved as the exams have appeared on the 'new spec' rather than being ill fitting old questions from previous iterations of the course.

I have been a qualified maths teacher teaching A Level and Further A Level Maths for 15 years and hope this book makes me millions so I can retire to a fishing village in Scotland with my family and dogs. I also hope this book helps you or a loved one become just a little more confident.

Thanks to my parents for being amazing for the last 45 years.

Thanks to my partner for helping me to live the life I want to.

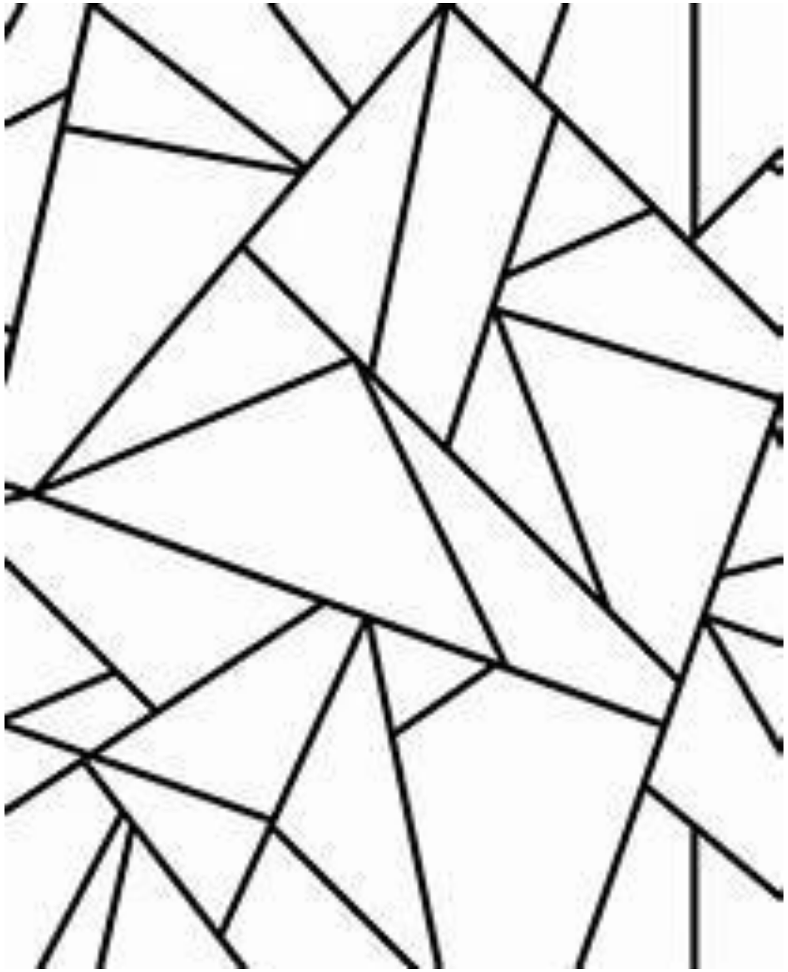
Thanks to my students who have (hopefully) proofread my questions and answers. That said, some minor errors may have slipped through!

Thank you to the Whippets and Iz for forcing me to get out every day and not being a socially awkward hermit.

I can’t write a book without showcasing my brother’s artwork. He works out of his studio and gallery (The Point in Cromer). His work is beautiful and can be found at www. richardkbladesartist.co.uk

Table of Contents

Indices, Surds and Expressions



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(1) Indices

WORKING AT D/E

(1) Simplify $6p^{\frac{1}{3}} \times 12p^{\frac{1}{2}}$

(2) Expand and simplify $x^2(2x - \frac{y}{x})$

(3) Simplify $8^{\frac{-1}{3}}$ without a calculator

WORKING AT B/C

(1) Simplify $\frac{2x^5+12x^{\frac{1}{5}}}{6x}$

(2) Solve the equation $25^{3-x} = 125^{x+1}$

(3) Without a calculator, simplify $\left(1\frac{9}{16}\right)^{-0.5}$

WORKING AT A*/A

(1) Write $\left(\frac{x^{\frac{1}{3}}}{16\sqrt{x}}\right)^{\frac{3}{4}}$ as a simplified power of x.

(2) Write $\left(\left(3x^{\frac{1}{2}}\right)^2 \times (4x^2)^2\right)^{\frac{1}{2}}$ in the form Ax^B where A is an integer and B is a rational fraction.

(3) The first and third terms of a geometric sequence are $2x^{\frac{2}{3}}$ and $8x^{\frac{16}{15}}$. What is the 2nd term?

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(2) Expanding Brackets

WORKING AT D/E

(1) Expand and simplify $(A+B)^2$.

(2) Without further expansion, use your answer to question (1) to find $(A-B)^2$

(3) Expand and simplify $(x+y)(2x-y+3)$

WORKING AT B/C

(1) Expand and simplify $-2x(3-x)^2$

(2) Expand and simplify $(3x+1)^2(3x-1)$

(3) Find the values of A, B and C such that $(2x+y)^3 \equiv Ax^3 + Bx^2y + Cxy^2 + y^3$

WORKING AT A*/A

(1) Expand and simplify $\left(x^{\frac{2}{3}} + x^{0.5}\right)^2$

(2) Find the terms independent of x in the expansion of: $(x+y)(4x-y)\left(y-\frac{3}{x}\right)$

(3) The two shorter sides of a right-angled triangle are $(x+1)^{\frac{1}{2}}$ and $(x-4)$. Find a simplified expression for the length of the remaining side in the form $(Ax^2+Bx+C)^N$ where A, B and C are integers and N is a simplified rational fraction.

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(3) Factorising Expressions

WORKING AT D/E

(1) Factorise $12x^2 + 19x + 4$ into double brackets.

(2) Factorise $121x^2 - 36$ by considering the difference of two squares.

(3) Show that $9x^2 + 6x + 1$ and be written in the form $(Ax + B)^2$

WORKING AT B/C

(1) Factorise $-4x^2 + 5x + 6$

(2) Fully factorise $20x^3 - 7x^2 - 3x$

(3) Show that $64x^4 - 25y^2$ can be written in the form $(Ax^n + By)(Ax^n - By)$ where A, B and n are integers to be found.

WORKING AT A*/A

(1) Fully Factorise $(3x + 1)^{31} - (3x + 1)^{30}$

(2) Fully factorise $169x - x^3y^2$

(3) Using the trigonometric identity (which you may know or will learn soon!)

$sin^2x + cos^2x \equiv 1$

show that $cos^4x - sin^4x \equiv cos^2x - sin^2x$

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(4) More Indices (Negative and Fractional)

WORKING AT D/E

(1) Without a calculator, find $\left(\frac{25}{36}\right)^{-\frac{1}{2}}$.

(2) Write $\sqrt[4]{x}$ in the form x^n

(3) Without a calculator, show that $\left(\frac{1}{8}\right)^{-\frac{2}{3}}$ is an integer.

WORKING AT B/C

(1) Write $x\sqrt{x}$ in the form x^n

(2) Fully simplify $\sqrt{\frac{32x^8}{2x^2}}$

(3) Write $\frac{(x^6)^{\frac{1}{3}}}{(x^4)^{-2}}$ in the form x^n where n is an integer to be found.

WORKING AT A*/A

(1) Without a calculator, simplify $\left(\frac{16}{81}x^{-0.25}\right)^{0.75}$

(2) Show that the fraction $\frac{(Ax+B)(Ax-B)}{A^2x^2}$ can be written as $1-\left(\frac{B}{Ax}\right)^2$

(3) Given that $P=\frac{27}{M^{12}}$ write $P^{\frac{-1}{3}}$ in terms of M. Give your answer as a simplified fraction.

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(5) Surds

WORKING AT D/E

(1) Without a calculator, show that $\left(\sqrt{2}+\sqrt{8}\right)^2$ is an integer. The common error is students writing 10.

(2) Without a calculator, show that $\frac{2}{3\sqrt{6}}$ can be written as $\frac{\sqrt{2a}}{3a}$ where a is an integer to be found.

(3) Without a calculator, show that $\frac{22}{5-\sqrt{3}}$ can be simplified to $5+\sqrt{3}$. Be rational here!

WORKING AT B/C

(1) Without a calculator, show that:

$$\left(\frac{1}{\sqrt{2}}+\sqrt{50}-\frac{\sqrt{2}}{2}-\sqrt{32}\right)^2 = 2$$

(2) Show that $\frac{\sqrt{6}+2}{\sqrt{6}-2}$ can be written as $\frac{\left(\sqrt{6}+2\right)^n}{n}$ where n is an integer

(3) Expand and simplify $\left(\sqrt{A}+\sqrt{B}\right)^2$

WORKING AT A*/A

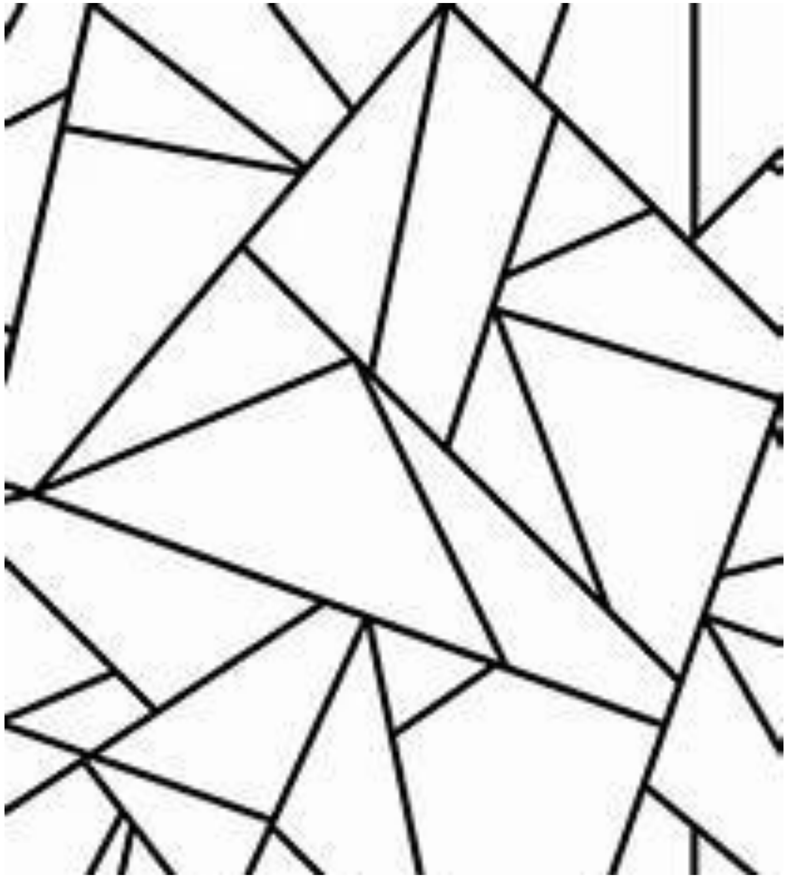
(1) Show that:

$$\left(\sqrt{A}+\sqrt{B}\right)^3 \equiv A^{\frac{3}{2}}+3AB^{\frac{1}{2}}+3BA^{\frac{1}{2}}+B^{\frac{3}{2}}$$

(2) Without a calculator, show that $\frac{20}{(2+\sqrt{2})(6-\sqrt{2})}$ can be written as $A(B-C\sqrt{C})$ where A is a rational fraction in its simplest form and B and C are integers.

(3) A rectangle has an area of $21+9\sqrt{3}$ and one side length of $\sqrt{3}+3$. Without a calculator, show that the perimeter of the rectangle can be written in the form $A\sqrt{B}+C$.

Quadratic Equations and Expressions



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(6) Solving Quadratic Equations

WORKING AT D/E

(1) How many solutions are there to the equation $x^2 = 1$?

(2) Solve the quadratic equation $8x^2 + 2x - 3 = 0$ by factorisation. Think double brackets.

(3) Solve the equation $x^2 - 4x - 8 = 0$ in the form $x = p \pm q\sqrt{r}$. You know this won't factor so you have two other choices.

WORKING AT B/C

(1) Without expanding the brackets, find the solutions to the equation $(4x-1)^2 = 25$

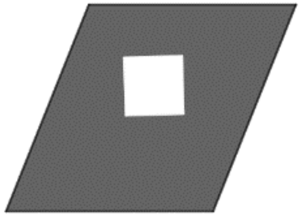
(2) Solve the equation $x - 4 - \frac{12}{x} = 0$ by first forming a quadratic equation.

(3) Solve the quadratic equation $4.9t^2 - t = 36$ giving each answer to 3SF

WORKING AT A*/A

(1) Find the only real solution to the equation: $\sqrt{x} - \frac{3}{\sqrt{x}} = 2,\ x > 0$

(2) The diagram below shows a parallelogram with a square removed. The base of the parallelogram is $(x+6)cm$ and the perpendicular height is $(13-x)cm$. The side length of the square is $(x+1)cm$. Given that the area of shaded part of the shape is $74cm$, find the least area of the white square.



(3) Show that the equation $4x = (8x-1)^{\frac{1}{2}},\ x > \frac{1}{8}$ has one solution.

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(7) Completing the Square for Quadratics Expressions

WORKING AT D/E

(1) Complete the square on the expression $x^2 + 6x - 8$

(2) By first factoring out the HCF, complete the square for $4x^2 - 8x$.

(3) Solve the quadratic equation $x^2 - 10x + 8 = 0$ giving your answer in the form $x = 5 \pm \sqrt{q}$ where q is a prime number. You must complete the square.

WORKING AT B/C

(1) Write the expression $x^2 - 5x + 1$ in the form $(x + p)^2 + r$

(2) Write the expression $-5x^2 + 10x + 7$ in the form $p(x + r)^2 + q$

(3) Show, by completing the square, that there are no real solutions to the equation $x^2 - 9x + 30 = 0$

WORKING AT A*/A

(1) By completing the square, solve the equation $2x^2 - 4px + 1 = 0$ giving your solutions in terms of p.

(2) By completing the square, find the maximum value of the function $f(x) = -x^2 - 3x + 8$ giving your answer as a rational fraction in its simplest form.

(3) Alan completes the square for a quadratic equation. He writes that $(2x - 3)^2 + 8 - k = 0$. He says there are two real roots to the equation. Explain why $k > 8$ for this to be true.

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(8) Function Notation

WORKING AT D/E

(1) $f(x) = 3 - x^2,\ x \in R.$

Find $f(1.5)$

(2) $g(x) = \frac{16}{x^2},\ x \in R.$

Given that $a < 0$ and that $g(a) = 4$, find the value of a.

(3) $m(x) = x^2 - 7$ and $n(x) = 3x + 3$.

Find the positive solution to $m(x) = n(x)$ by setting them equal to each other.

WORKING AT B/C

(1) $f(x) = x^3 - 4x,\ x \in R,$

Find the roots of $f(x)$

(2) $g(x) = x^2 + 12x,\ x \in R,$

$g(x)$ has a minimum value of q when $x = p$. Find the values of p and q.

(3) $f(x) = x^3 - 7$ and $g(x) = x(x+1)(x-2)$

Find the solutions to $f(x) = g(x)$ giving your answers as simplified surds.

WORKING AT A*/A

(1) $f(t) = t^{-1.5} + 1$

Given that $f(a) = 28$, find the value of a

(2) $m(x) = x^6 + 7x^3 - 8, x \in R,$

Show that the roots of $m(x)$ are integers.

(3) $h(x) = (x+1)^2(x^2-3)\ \ x \in R,$

Write down the roots of $h(x)$ in ascending order.

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(9) Sketching Quadratic Graphs

WORKING AT D/E

(1) Sketch the graph of $y = x^2 - 4x - 12$ showing where the graph crosses the coordinate axes.

(2) Sketch the graph of $y = -x^2 + 12$ showing the roots of the equation in the form $x = \pm p\sqrt{q}$

(3) By completing the square, sketch the graph of $y = x^2 - 2x + 4$, showing the coordinates of the minimum point.

WORKING AT B/C

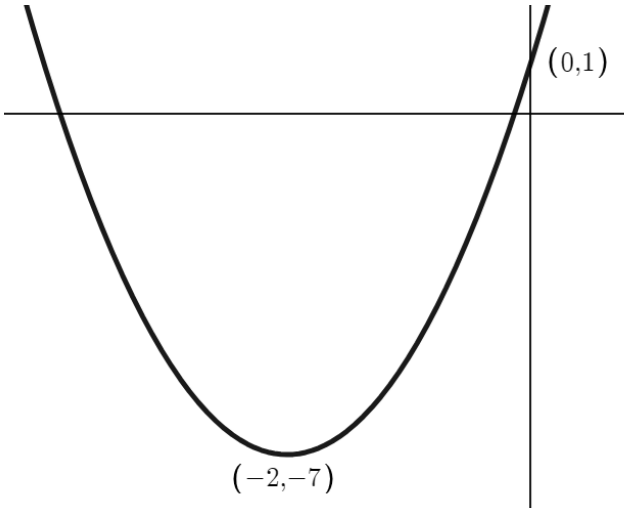
(1) Sketch the graph of $y = x^2 - 4x - 12$ showing the roots, the y intercept and the minimum point.

(2) Sketch the graph of $y = -x^2 + 6x + 12$ showing the equation of the axes of symmetry and the coordinates of the turning point. State whether the turning point is a maximum or minimum.

(3) Sketch the graph of $y = 5x^2 - 10x + 1$ showing the coordinates of the minimum point and the roots of the equation.

WORKING AT A*/A

(1) The graph of $y = 2x^2 + bx + c$ is shown below. The points $(0,1)$ and $(-2,-7)$ lie on the curve.



Find the roots of the equation in the form:

$x = p \pm r\sqrt{q}$

(2) Sketch the graph of $y = -7x^2 + 10x + 1$, showing the coordinates of the turning point and any points where the graph crosses the coordinate axes.

(3) Given that the graph of $y = x^2 + px + q$ doesn't touch or cross the x axis, show that $p^2 < 4q$

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(10) The Discriminant for Quadratic Equations

WORKING AT D/E

(1) Complete the sentence:
"If $b^2 - 4ac < 0$ then there are…

(2) State the number of real roots to the equation $x^2 + 6x + 5 = 0$ by considering the discriminant.

(3) Sketch the graph of a quadratic equation that has a discriminant of 0.

WORKING AT B/C

(1) The equation $x^2 + kx + 16 = 0$ has a repeated real root. Find the two possible values of k.

(2) The quadratic equation $6x^2 + 4kx + 5 = 0$, $k < 0$ has a discriminant of -56. Find the value of k.

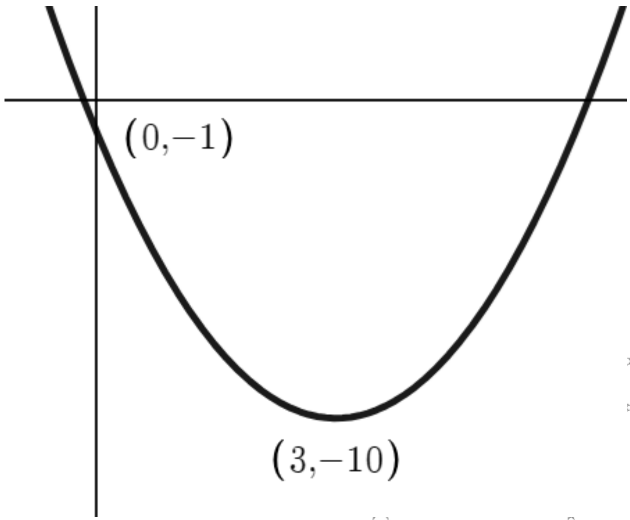
(3) The quadratic equation $kx^2 + 5kx = 3$ has no real roots. Find the set of values that satisfy k.

WORKING AT A*/A

(1) The graphs of $y = 3$ and $y = x^2 + kx + 10$ do not intersect. Show that $-2\sqrt{7} < k < 2\sqrt{7}$

(2) The equation $4kx^2 + 4kx + 4 = 0$, $k \neq 0$ has a repeated root. Find the numeric value of this root.

(3) The diagram below shows part of the graph of $y = x^2 + px + q$. The points $(0, -1)$ and $(3, -10)$ lie on the curve. Find the value of the discriminant for $x^2 + px + q = 0$.



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(11) Applications of Quadratics Equations

WORKING AT D/E

(1) The velocity (V) of a toy car after (t) seconds is given by $V = -t^2 + 8t + 3$ for $0 \leq t \leq 3$

(a) Find the initial velocity of the toy car

(b) Find the velocity of the toy car after 2 seconds.

(c) Show that the car is never stationary.

WORKING AT B/C

(1) The height in metres (h) of a wave produced by a wave machine in a swimming pool over time (t) seconds is modelled by the equation $h = -t^2 + 10t$ for $t \geq 0$

(a) State the initial height of the wave.

(b) Find to 3SF when the wave is first 18m high.

(c) Find the maximum height of a wave,

(d) State, with a reason, the values of t for which the model would be valid.

WORKING AT A*/A

(1) A driver stands on a 5-metre platform and performs a dive into a swimming pool below. The height the diver above the water is modelled by the equation $h = -2t^2 + 2t + k$ where h is the height in metres above the water and t is the number of seconds from when the dive is performed.

(a) State the value of k

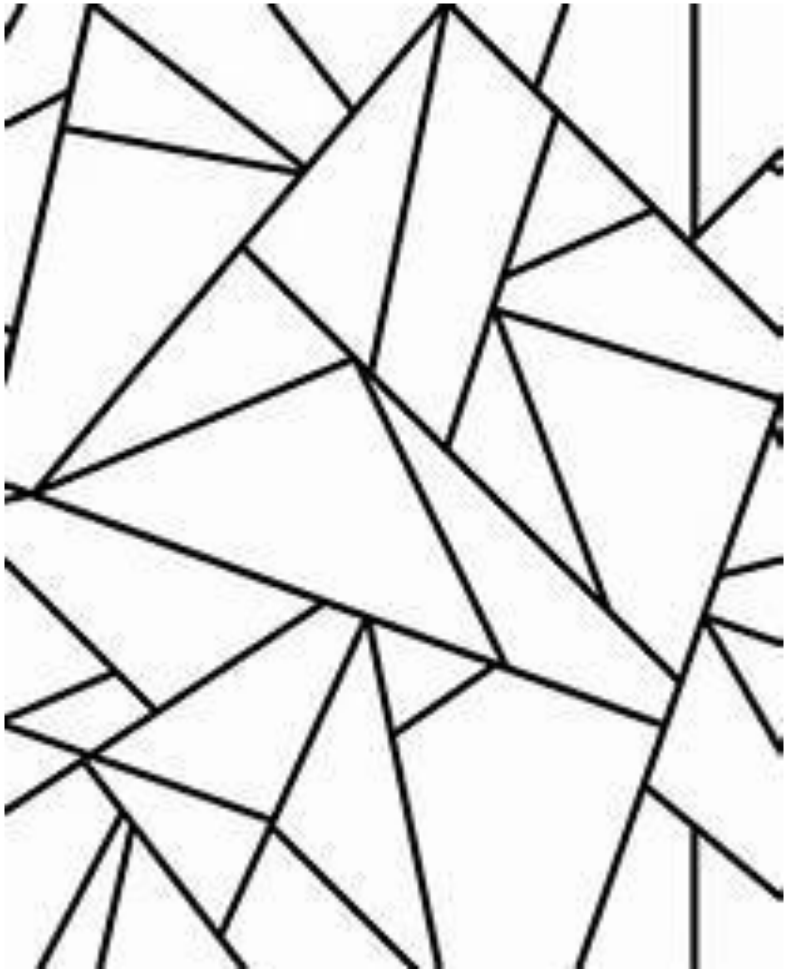
(b) Find to 3SF the time the diver hits the water.

(c) How long does it take the diver to reach their maximum height and what maximum height did they reach?

(d) Explain why the model may no longer be valid after the diver hits the water.

(e) Sketch the graph for the model.

Simultaneous Equations & Inequalities



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(12) Solving Linear Simultaneous Equations

WORKING AT D/E

(1) Solve the simultaneous equations:

$$2x + 5y = 14$$
$$3x - 2y = 2$$

(2) Solve the simultaneous equations:

$$y = 2x + 4$$
$$y = 7x - 11$$

(3) Show there are no solutions to the simultaneous equations

$$y = 3x + 6$$
$$y = 3x - 1$$

WORKING AT B/C

(1) Solve the simultaneous equations:

$$0.1x + 4y = 9$$
$$0.3x - y = 1$$

(2) Solve the simultaneous equations:

$$3y = 6 - 3x$$
$$2y = 5x - 3$$

(3) The line simultaneous equations below have no solutions:

$$3x + py = 14$$
$$2x - 7y + 9 = 0$$

Given that p is a constant, find the value of p.

WORKING AT A*/A

(1) A square has side lengths $x + y$ and a perimeter of 24cm. A rectangle has side lengths of y and $x + 2y$. Its perimeter is 2/3rds that of the square. How much larger is the area of the square than the area of the rectangle?

(2) The linear simultaneous equations:

$$qx + py = 26$$
$$4x - y + q = 0$$

have the solutions $x = 0.5p$ and $y = 7$. Find the integer values of the constants p and q.

(3) Write a pair of linear simultaneous equations that have no solutions.

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(13) Linear & Non-Linear Simultaneous Equations

WORKING AT D/E

(1) Solve the simultaneous equations

$$y = x^2 + 5x - 13$$

$$y = 2x - 3$$

(2) Solve the simultaneous equations using the method of substitution

$$y = x - 1$$

$$xy = 6$$

(3) Solve the simultaneous equations

$$y = x$$

$$x^2 + y^2 = 72$$

WORKING AT B/C

(1) Solve the simultaneous equations

$$2y + x = 10$$

$$xy = 8$$

(2) Solve the simultaneous equations

$$3y + 5x = 7$$

$$x^2 + y^2 = 5$$

(3) Show that there are no solutions to the simultaneous equations

$$y = 3$$

$$(x - 8)^2 + (y + 7)^2 = 4$$

WORKING AT A*/A

(1) A circle with centre (0,0) and radius $5\sqrt{2}$ and a line with gradient -1 passing through (0,0) meet at the points (a, b) and (c, d) where $a < c$. Find the values of a, b, c and d.

(2) Solve the simultaneous equations

$$xy = 2y^2 - 30$$

$$2x + 3y = 13$$

Giving any non-integer answers as exact fractions in their simplest form.

(3) A square has side length p and area of q. Given that the perimeter of the square is $\frac{2q}{3}$, show that the length of the diagonal of the square can be written in the form $a\sqrt{b}$ where a and b are integers to be determined.

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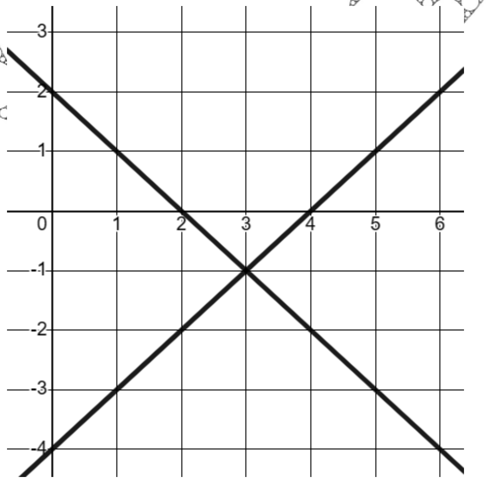
(14) Graphing Simultaneous Equations

WORKING AT D/E

(1) The diagram below shows the graphs of

$y = 2 - x$ and $y = x - 4$



<u>Use the graphs</u> to solve the simultaneous equations

$$y = 2 - x$$
$$y = x - 4$$

(2) By sketching the graphs of $y = x^2 + 2$ and $y = 1 - x^2$ on the same set of axes, show that there are no solutions to the simultaneous equations

$$y = x^2 + 2$$
$$y = 1 - x^2$$

WORKING AT B/C

(1) Sketch the graphs of $y = 4 - x^2$ and $x + y = 3$ on the same set of axes to find the number of solutions to the simultaneous equations:

$$y = 4 - x^2$$

$$x + y = 3$$

(2) (a) Sketch the graphs of $x^2 + y^2 = 50$ and $y = -x$ on the same set of axes.

(b) Use algebra to shows the points of intersection of the two graphs have integer solutions.

(3) By considering the discriminant, state the number of times the graphs of $x^2 - y = 3$ and $y = 5 - x$ meet or intersect.

WORKING AT A*/A

(1) The graph of $x^2 + y^2 = 30$ has a tangent with equation $y = 2x + k$ where k is a constant. Show that $k = \pm 5\sqrt{6}$

(2) The graphs of $y = 3x^2 + k$ and the line with equation $y = mx$ where k and m are constants do not intersect. Explain clear why:

$$-2\sqrt{3k} < m < 2\sqrt{3k}$$

(3) The height (h) metres of a rocket above the launch pad after (t) seconds can be modelled by the equation $h = -2t^2 + kt$ where k is a constant and $t \geq 0$. Find the value of k such that the maximum height of the rocket is 30 metres above the launch pad. Given your answer in exact form.

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(15) Linear Inequalities

WORKING AT D/E

(1) Solve the inequality:

$$3x - 4 < 8 - x$$

(2) (a) On a number line draw represent the following **two** inequalities individually:

$2x < 10$ and $x \leq -1$

(b) Hence, write down the integers that satisfy both $2x < 10$ and $x \leq -1$

(3) Solve the inequality:

$$-0.1x \leq 5$$

WORKING AT B/C

(1) Solve the inequality:

$$3 - \frac{4x}{2} > -3$$

(2) Solve the inequation $x(x-1) < x^2 - 8$

(3) Find the set of values of x that satisfy both

$3 - 6x \leq 0$ and $-8 < 2x + 14 < 24$

WORKING AT A*/A

(1) Given that there are no values that satisfy BOTH

$2kx \leq 1$ and $3(4x - 8) \geq x$

Find the set of values for the positive constant k

(2) Given that k is a negative constant, find the set of values of x such that $6 \leq kx + 1 < 10$ giving your answer in terms of k.

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(16) Quadratic Inequalities

WORKING AT D/E

(1) Find the set of values that satisfy

$$x^2 - x - 6 < 0$$

(2) Find the set of values that satisfy

$$x^2 + 8x + 12 \geq 0$$

(3) Find the set of values that satisfy

$$x^2 \geq 4$$

WORKING AT B/C

(1) Find the set of values that satisfy $-x^2 < x - 12$

(2) Find the set of values that satisfy both

$$6x^2 \leq 17x + 3 \text{ and } 4 \geq 2x$$

(3) Find the set of values that satisfy both

$$x^2 \leq 12 \text{ and } x^2 > x$$

Give your answer as an inequality in exact form.

WORKING AT A*/A

(1) Given that there are no common values of x that satisfy both $6x^2 + x - 1 \leq 0$ and $2x > k$ where k is a constant, find the set of values of k.

(2) Find the set of values of x such that $\frac{4}{x} > 2$

(3) Find the set of values such that $\frac{8}{x+1} \leq 1$

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(17) Graphing Inequalities

WORKING AT D/E

(1) $f(x) = 4$ and $g(x) = x^2$

(a) Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same set of axes.

(b) Find the coordinates where the graphs of $f(x)$ and $g(x)$ meet.

(c) Hence, find the values of x for which $f(x) > g(x)$

WORKING AT B/C

(1) $f(x) = 32 - x^2$ and $g(x) = x^2$

(a) Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same set of axes.

(b) Find the coordinates where the graphs of $f(x)$ and $g(x)$ meet.

(c) Hence, find the values of x for which $f(x) \leq g(x)$

WORKING AT A*/A

(1) $f(x) = 28 - x$ and $g(x) = x^2 + k$, $0 < k < 28$

(a) Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same set of axes.

(b) Given that the set of values for which $f(x) > g(x)$ is $-5 < x < 4$, find the value of k.

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(18) Shading Inequalities

WORKING AT D/E

(1) (a) Sketch the lines $x = 2$ and $y = x$ on the same set of coordinate axes.

(b) Hence, shade the region where $x < 2$ and $x \geq y$

(2) (a) Sketch the graphs of $y = 3$ and $y = 2x^2$ on the same set of coordinate axes.

(b) Hence, shade the region where $2x^2 < 3$

WORKING AT B/C

(1) (a) Sketch the graphs of $x + y = 6$ and $y = 10 - x^2$ on the same set of coordinate axes.

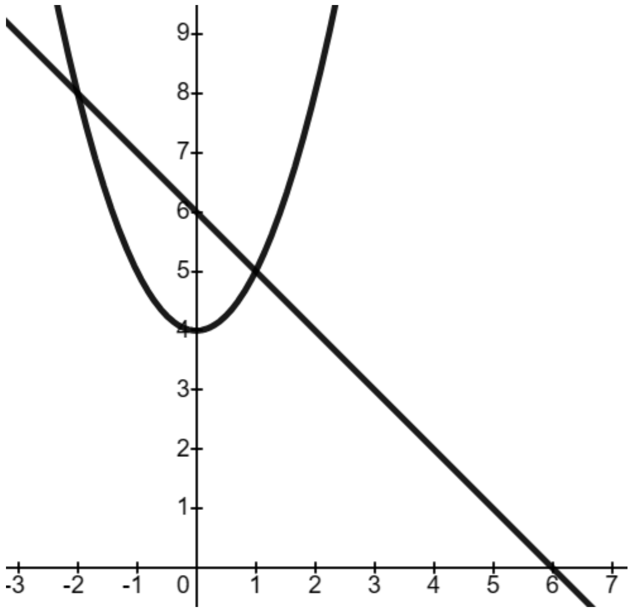
(b) **Hence**, shade the region where $6 - x < 10 - x^2$

(2) Shade the region on a graph where $x + 5 < x^2$

(3) By sketching two different graphs, show that there is no region that satisfies $x^2 + 5 < \frac{1}{4}x - 3$

WORKING AT A*/A

(1) The diagram below shows the graph of $y = x^2 + a$ and the graph of $y = b - x$.



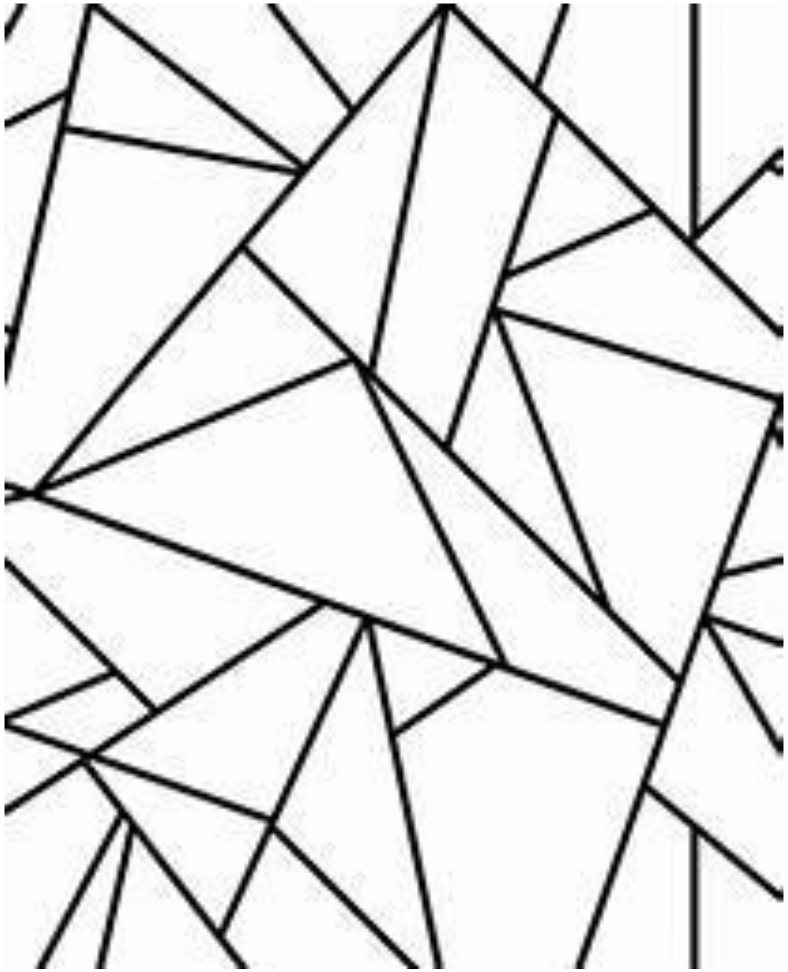
(a) Write down the value of the constants a and b.

(b) **Using your answer to part (a)**, on the graph above, shade the region that satisfies:

$$x^2 + x - 2 \leq 0$$

You must show full workings.

Graph Sketching



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(19) Cubic Graphs

WORKING AT D/E

(1) Sketch the graph of $y = (x - 1)(x + 2)(x - 5)$ showing where the curve crosses the coordinate axes

(2) (a) Show that $x^3 + 2x^2 - 8x$ can be written as $x(x - a)(x - b)$ where a and b are integers.

(b) Hence, sketch the graph of $y = x^3 + 2x^2 - 8x$

(3) Sketch the graph of $y = x(2 + x)(3 - x)$

WORKING AT B/C

(1) (a) Write $x^3 + 4x^2 + 4x$ in the form $x(x + a)^2$ where a is an integer to be found.

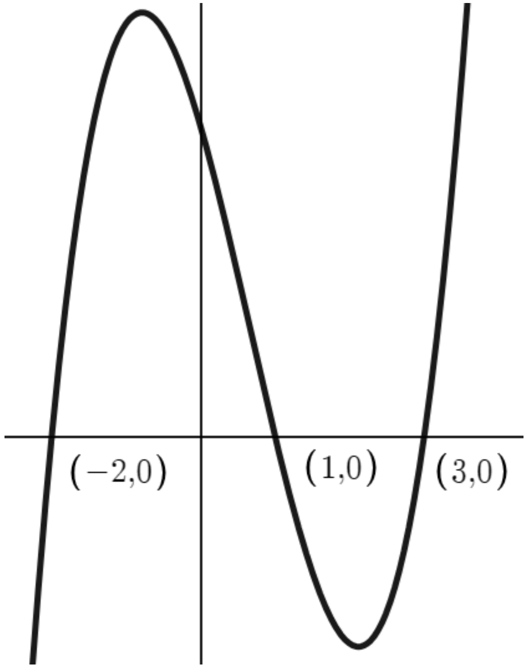
(b) Hence, sketch the graph of $y = x^3 + 4x^2 + 4x$ showing and points where the curve meets or crosses the coordinate axes.

(2) Sketch the graph of $y = -x^3 + x$ showing where the curve crosses the coordinate axes.

(3) Sketch the graph of $y = (2x - 1)^3$

WORKING AT A*/A

(1) The diagram below shows part of the curve with equation $y = 2x^3 + bx^2 + cx + d$



Find the values of the constants b, c and d.

(2) Sketch the graph of $y = x^3 + ax$ where a is a constant and $a > 0$.

(3) Sketch the curve of $y = -ax^3 + bx$ where a and b are positive constants.

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(20) Quartic Graphs

WORKING AT D/E

(1) Sketch the graph of

$y = (x-1)(x+2)(x-3)(x-5)$ showing where the curve crosses the coordinate axes.

(2) Sketch the graph of

$y = (x+3)(x-2)(x+6)(3-x)$ showing where the curve crosses the coordinate axes.

(3) Sketch the graph of

$y = (x+2)^4$ showing where the curve crosses the coordinate axes.

WORKING AT B/C

(1) Sketch the graph of

$y = -x(x+2)(x-3)(x-5)$ showing where the curve crosses the coordinate axes.

(2) (a) Show that $x^4 - x^2$ can be written as $x^2(x+1)(x-1)$

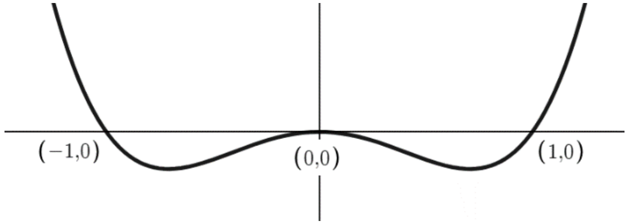
(b) Hence, draw the graph of $y = x^4 - x^2$ showing where the curve meets or crosses the coordinate axes.

(3) Sketch the graph of

$y = (3x+1)(x-1)(3-x)^2$

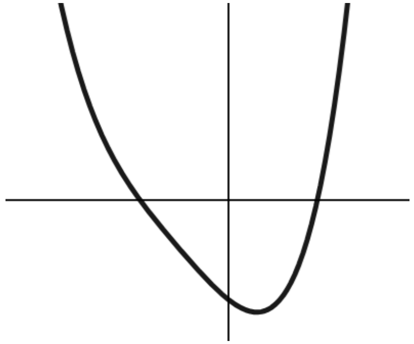
WORKING AT A*/A

(1) The diagram below shows part of the graph with equations $y = x^4 + bx^3 + cx^2 + dx + e$



Find the values of the constants b, c, d and e.

(2) The diagram below shows part of a graph of a quartic equation. All the roots to the equation are shown.



Write a **possible** equation for the graph.

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(21) Reciprocal Graphs

WORKING AT D/E

(1) Draw the graph of $y = \frac{1}{x}$ including the equations of any asymptotes.

(2) Draw the graph of $y = \frac{1}{x^2}$ including the equations of any asymptotes.

(3) Draw the graph of $y = -\frac{1}{x}$ including the equations of any asymptotes.

WORKING AT B/C

(1) (a) On the same set of axes, sketch the graphs of $y = \frac{2}{x}$ and $x + y = 6$

(b) Hence, state the number of solutions to the simultaneous equations:

$$y = \frac{2}{x}$$

$$x + y = 6$$

(2) By draw two different graphs, show that there are 2 real solutions to the simultaneous equations

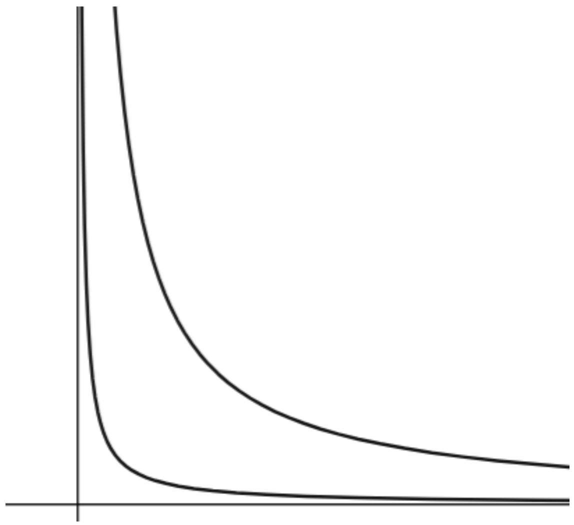
$$y = \frac{1}{x^2}$$

$$y = 6$$

(3) Write down the equations of the asymptotes of the curve $y = 8x^{-2}$

WORKING AT A*/A

(1) The diagram below shows part of the curves of $y = \frac{a}{x}$ and $y = \frac{b}{x}$ where a and b are positive constants and $b > a$.



Label each graph with its equation.

(2) The graph of $y = \frac{a}{x^2}$ passes the point $(-2, -16)$

(a) Find the value of a

(b) Sketch the graph of $y = \frac{a}{x^2}$ showing any asymptotes on the graph.

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(22) The Intersection of Graphs

WORKING AT D/E

(1) (a) On the same set of axes, draw the graphs of $x^2 + y^2 = 1$ and $y = x + 5$

(b) Write down how many points of intersection there are of the two graphs.

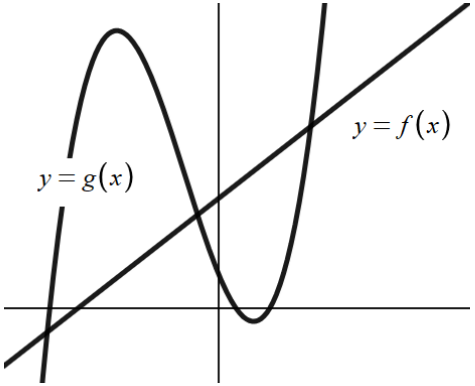
(2) By drawing the graphs of $y = x^2$ and $y = 2x$, state the number of solutions to the simultaneous equations:

$$y = x^2$$

$$y = 2x$$

WORKING AT B/C

(1) The diagram below shows the cubic function $g(x)$ and the linear function $f(x)$



Beryl is a maths student and she says there are 4 real solutions to the equations $f(x) = g(x)$. Explain why she is wrong.

(2) **By drawing two graphs**, state the number of real solutions to the simultaneous equations

$$y = 8 - x^3$$

$$y = 2x^2$$

(3) **By drawing two graphs**, state the number of real solutions to the simultaneous equations

$$y = (x + 2)(x - 3)(x - 5)(x - 7)$$

$$y = 0$$

WORKING AT A*/A

(1) (a) On the same set of axes, draw the graphs of $y = x^3 - 3x^2$ and $y = 8 - 3x^2$

(b) Explain why there are no points of intersection when $x < 0$.

(2) (a) On the same set of axes, draw the graphs of $y = ax^2$ and $y = \frac{a}{x}$ where a is a positive constant.

(b) Find the coordinates of any points where the graphs meet. Give your answer(s) in terms of a

(3) What is the maximum number of real solutions to the equation $f(x) = g(x)$ if $f(x)$ is a cubic function and $g(x)$ is a quartic function? You must explain your answer fully.

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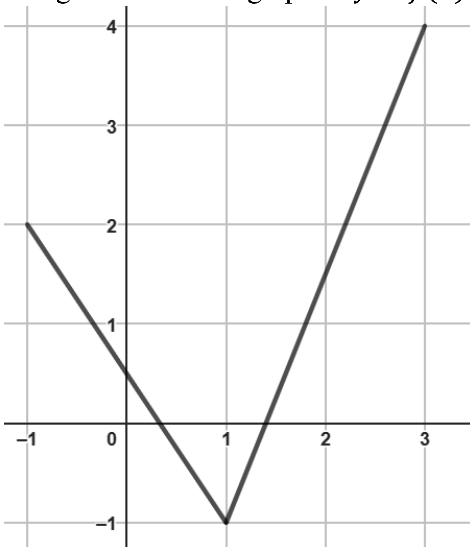
(23) Transforming Graphs (Translations)

WORKING AT D/E

(1) (a) Sketch the graph of $y = x^3$
(b) Hence sketch the graph of $y = (x+5)^3$
(c) Hence sketch the graph of $y = x^3 + 3$

(2) $f(x) = (x-2)^2 + 1$
(a) Sketch $y = f(x)$
(b) Sketch $y = f(x+2)$
(c) Sketch $y = f(x) - 4$

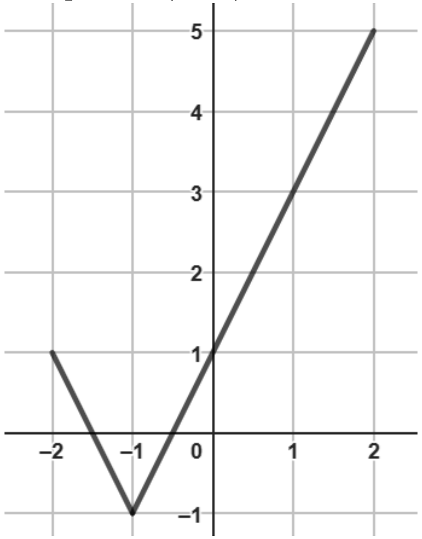
(3) The diagram shows the graph of $y = f(x)$



Draw the graph of $y = 1 + f(x-2)$

WORKING AT B/C

(1) The diagram shows the graph of $y = h(x)$ with the minimum point at $(-1,1)$



The graph of $y = h(x)$ is translated to the graph of $y = g(x)$. The minimum point of $g(x)$ has coordinates $(2,1)$. State fully the transformation that maps $h(x)$ to $g(x)$

(2) (a) State the single transformation that maps the graph of $y = \frac{1}{x}$ to the graph of $y = \frac{1}{x-3}$
(b) **Hence** sketch the graph of $y = \frac{1}{x-3}$ showing where the curve crosses the axes and any asymptotes on the curve,

(3) The graph of $y = x^2$ to translated by the vector $\binom{3}{4}$. Write the equation of the newly translated graph in the form $y = ax^2 + bx + c$

WORKING AT A*/A

(1) $f(x) = \frac{1}{x^2}$ is translated by the vector $\binom{-2}{-1}$ to give $g(x)$. Sketch the graph of $y = g(x)$ showing any points where the graph crosses the coordinate axes and label any asymptotes.

(2) Given that $f(x) = x(x-1)(x+2)$ and that $g(x) = (x-3)(x-4)(x-1)$ state the single transformation that maps the graph of $y = f(x)$ to $y = g(x)$

(3) $f(x) = x^2 - 4x - 10$

The graph of $y = f(x)$ is transformed to the graph of $y = f(x) + a$. Given that there are no real solutions to the equation $f(x) + a = 0$ find the set of values of a.

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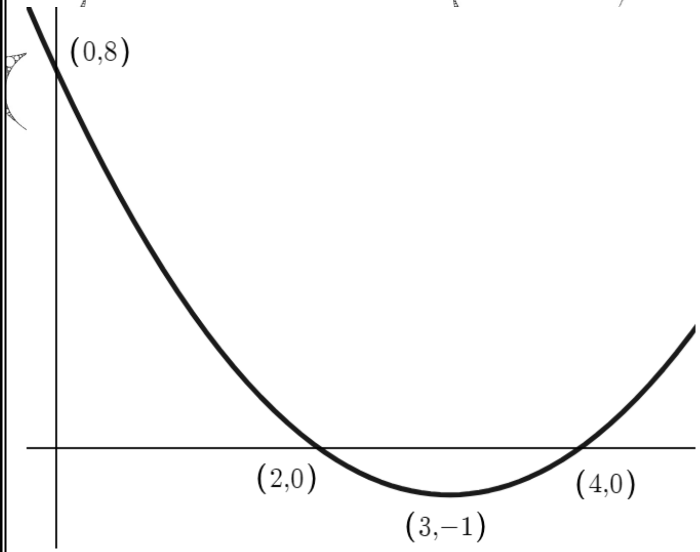
(24) Transforming Graphs (Stretching/Reflecting)

WORKING AT D/E

(1) The graph of $y = f(x)$ is shown below.



Sketch the graphs of the following showing the coordinates on each graph:

(a) $y = 3f(x)$ (b) $y = f(2x)$ (c) $y = -f(x)$

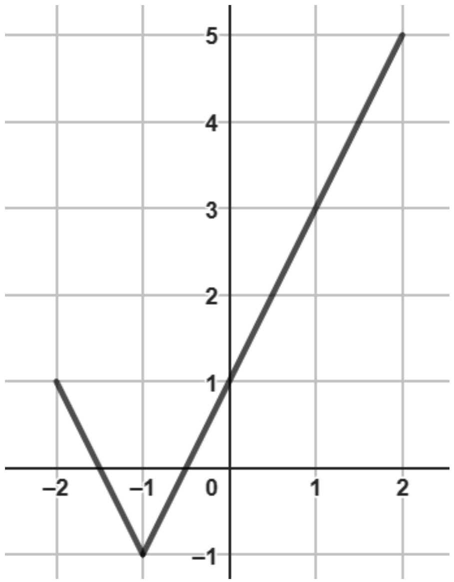
(d) $y = f(-x)$ (e) $y = f(0.5x)$ (f) $y = -4f(x)$

(g) $y = -f(-x)$ (h) $y = 0.5f(x)$ (i) $y = 1 - f(x)$

WORKING AT B/C

(1) The graph of $y = f(x)$ is transformed to the graph of $y = 5f(x-1)$. State fully the transformations that map the graphs of $y = f(x)$ to $y = 5f(x-1)$.

(2) The graph of $y = g(x)$ is shown below. The minimum point has coordinates $(-2, -1)$.



Sketch the graph of $y = -2g\left(\frac{x}{2}\right)$ stating the coordinates of the maximum point.

(3) (a) Sketch the graph of
$y = (x-4)(x+8)(x-12)(x-1)$
(b) **HENCE** Sketch the graph of
$y = (4x-4)(4x+8)(4x-12)(4x-1)$

WORKING AT A*/A

(1) $f(x) = x^2 - 6x + 9$ and
$g(x) = 4x^2 - 12x + 9$

State **fully**, the transformations that maps the graph of $y = f(x)$ to the graph of $y = g(x)$.

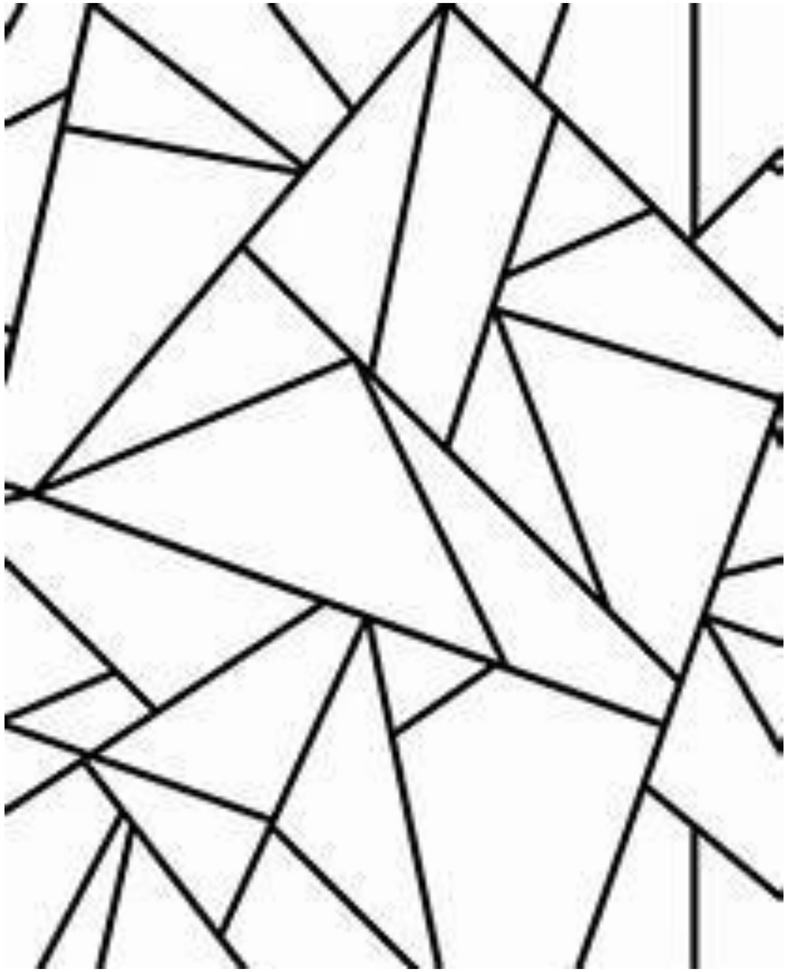
(2) $h(x) = (x-2)^2(x-4)$

The graph of $y = h(x)$ is transformed to the graph of $y = kh(x)$ where k is a constant. Given that the graph of $y = kh(x)$ crosses the y axis at the point $(0,24)$ find the value of k.

(3) Sketch $y = 3(x-2)(x+1)(x-a)^2$ where $a > 2$

Show where the graph meets or crosses the coordinate axes giving your answers in terms of a where appropriate.

Straight Line Graphs $y = mx + c$



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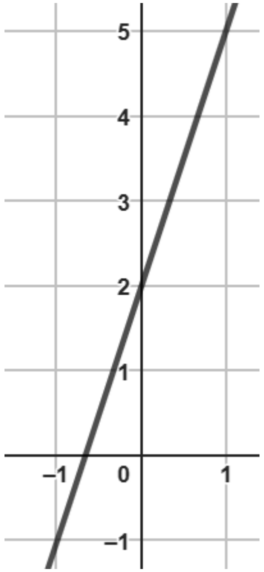
(25) Straight Line Graphs in the form $y = mx + c$

WORKING AT D/E

(1) Find the gradient of the line passing through the points $(-1,8)$ and $(4,10)$ giving your answer as a simplified fraction.

(2) (a) Write down the equation of the line shown in the form $y = mx + c$



(b) Draw the line with equation $y = 3 - x$

(3) What is the gradient of the line with equation $6x + 4y = 3$?

WORKING AT B/C

(1) A line passing through the points $(-6, p)$ and $(2, -4)$ has gradient $-\frac{9}{8}$..

(a) Find the value of p

(b) Find where the line crosses the coordinate axes.

(2) A line with gradient $\frac{3}{5}$ passes through the point $(8,2)$.

(a) Find the equation of the line in the form $ax + by = c$

(b) The line passes through the point $(0, q)$. Show that q is a rational fraction.

(3) The line with equation $ax + 10y - 2 = 0$ has a gradient of $\frac{4}{7}$. Find the value of a.

WORKING AT A*/A

(1) The line $ax + by - 40 = 0$ where a and b are integers in their simplest form. Given that passes through the coordinate axes at $(10,0)$ and $(0,20)$, find the values of a and b.

(2) The line with equation $ax + by + c = 0$, passes through the positive x axis. Given that a is negative and b and c are positive:

Write an inequality in x in terms of a and c

(3) L_1 has equation $ax + by + c = 0$ and L_2 has equation $y = px + q$. Given that the lines do not intersect, and they are NOT the same straight line, show that $a + bp = 0$

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(26) More Straight Line Graphs

WORKING AT D/E

(1) The lines with equation $y = 2x + 1$ and $4x - y - 8 = 0$ meet at the point (a, b). Find the values of a and b.

(2) The equation of a straight line is given as $5x + 10y = 20$. Write the equation of the line in the form $y = mx + c$

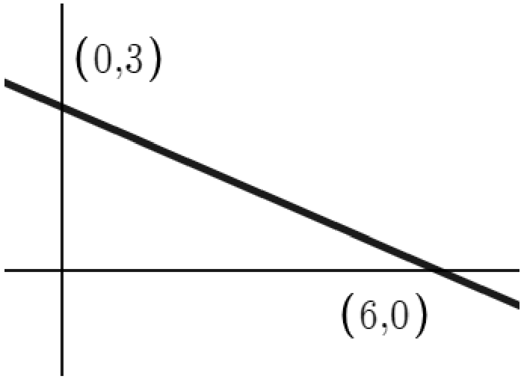
(3) L_1 has equation $6x - 8y + 8 = 0$. Find where the line cuts the coordinate axes.

WORKING AT B/C

(1) L_1 has equation $y = px - 8$ and L_2 has equation $y = 5x - 12$. L_1 and L_2 intersect at the point $(-4, q)$.

Find the values of the constants p and q.

(2) Find the equation of line shown below in the form $ax + by + c = 0$



(3) $f(x) = 2x + 1$

Find where the graph of $y = f(x - 3)$ crosses the x axis.

WORKING AT A*/A

(1) The line with equation $ax + by = c$ crosses the coordinate axes at A and B. Show that the area of the triangle AOB where O is the origin can be written as $\frac{c^2}{2ab}$

(2) The line with equation $px + qy = r$ and the line with equation $y = mx + c$ intersect on the y axis. Show that $r = cq$

(3) A line with gradient $\frac{-2}{3}$ passes through the points $(p, 0)$ and $(0, q)$ where p and q integers. Find the least possible positives values of p and q.

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(27) Straight Line Graphs (Parallel & Perpendicular)

WORKING AT D/E

(1) Write down gradient of the line (a) parallel and (b) perpendicular to the line with equation $y = 3x$

(2) Show that the lines $5x + 2y = 8$ and $y - x = 4$

Are neither parallel nor perpendicular.

(3) Find the equation of the line perpendicular to the line with equation $y = \frac{2}{5}x - 8$ that passes through the point $(2,3)$. Give your answer in the form $y = mx + c$

WORKING AT B/C

(1) Given that the line with equation $y = 4x + 7$ is perpendicular to the line with equation $ax - 2y + 8 = 0$, show that $a = \frac{-1}{2}$

(2) A $(-1,5)$ and B $(5,1)$ create the line segment AB. Show, using algebra, that the perpendicular bisector of AB can be written in the form $y = mx$ where m is a constant to be found.

(3) The line $y = px + c$ is parallel to a line passing through the points $(a, 0)$ and $(0, b)$. Write an expression for p in terms of a and b.

WORKING AT A*/A

(1) The perpendicular bisector of the line $x + y = a$ where a is a positive constant has equation $py = qx + r$ where p, q and r are also constants.

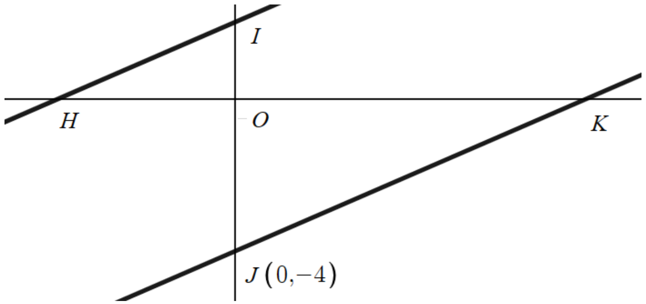
Show, with full workings, that $p = q$ and that $r = 0$

(2) Lines L_1 and L_2 are two different lines.

The equation of L_1 is $y = mx + c$
The equation of L_2 is $x + py + q = 0$
Where m, p and q are non-zero constants.

Find the set of values for p in terms of m for which the lines intersect.

(3) The diagram below shows two parallel lines. The points H, I, J $(0,4)$ and K lie on one of the two lines.



Given O is the origin, $OJ = 2OI$ and $OH = 2.5OI$, find the equation of the line passing through the points I and K in the form $y = mx + c$

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(28) The Geometry of Straight Lines

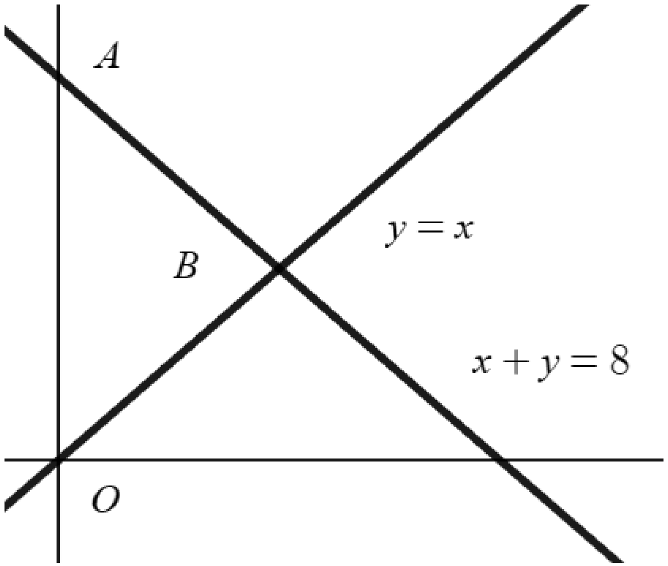
WORKING AT D/E

(1) Find the length of the line segment from $(-1,8)$ to $(7,2)$

(2) The line with equation $x+y=6$ crosses the x axis at A and the y axis at B. Find the area of the triangle AOB where O is the origin.

WORKING AT B/C

(1) The diagram below shows the graphs of $y=x$ and $x+y=8$. The lines meet at the point B. Points A and O are where the two lines meet the y axis.



Find the area of ΔAOB

(2) The length of the line segment AB is $4\sqrt{2}$. Given that the coordinates of A and B are $(4,-1)$ and $(8,p)$ respectively, find the possible values of p

(3) The line $y=\frac{5}{2}x-10$ crosses the coordinate axes at A and B. Find the length of the line AB as a simplified surd.

WORKING AT A*/A

(1) The perpendicular bisector of the line through the points $(-11,8)$ and $(6,4)$ crosses the coordinate axes at A and B. Find the area of triangle AOB where O is the origin. Give your answer in exact form.

(2) A line of gradient 1 passes through the points A $(3,4)$ and B (p,q). Given that the length $AB=6$, find the possible values of p and q giving your answers in surd form.

(3) The lines with equations $x=6$ and $y=2x+c$ enclose a trapezium of area 48 between the two lines, the positive x axis and the positive y axis. Find the value of c.

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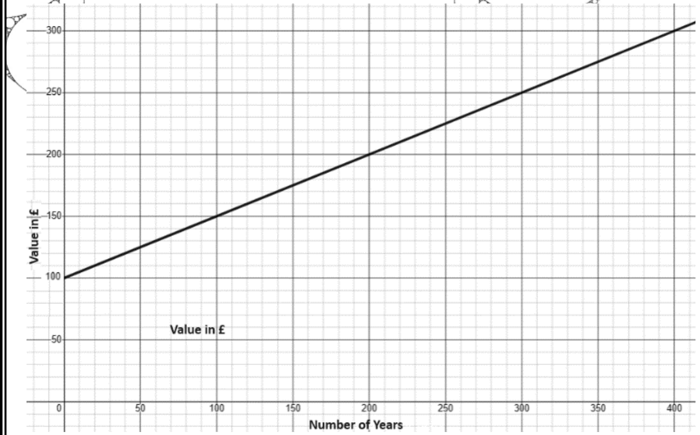
(29) The Application of Linear Graphs

WORKING AT D/E

(1) The diagram below shows **a very basic** model of the value of the painting from when it was first sold.



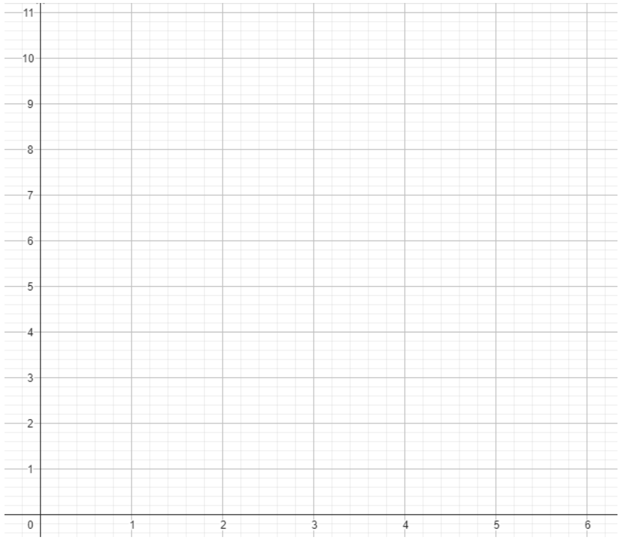
(a) Interpret the value of 100 on the vertical axis.
(b) Find the gradient of the line.
(c) Explain what the gradient represents in context of the model.
(d) Hence write an equation for the model in the form $V = aN + b$ where V is the value of the painting in £ and N is the number of years after the painting was first sold.

WORKING AT B/C

(1) The table below shows the length (L) of a genetically modified leaf in cm over a number of weeks (W).

W	0	1	2	3	4	5	6
L	0	1.8	3.6	5.4	7.2	9	10.8

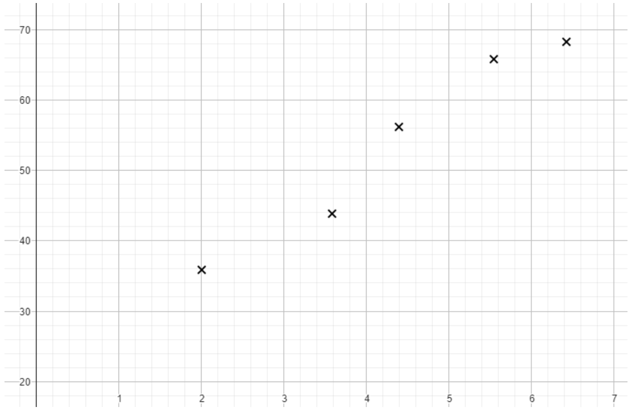
(a) Plot the points on a graph like that below and connect them.



(b) Is the data suitable for a linear model?
(c) Explain why the model is an example of direct proportion.
(d) Write an equation for the length of the leaf in the form $L = aW + b$
(e) Interpret, in context, the constant a and explain, in context, why $b = 0$.
(f) Explain the long-term possible limitations of the model.
(g) Find how many weeks it will take for the leaf to have a length of 37cm

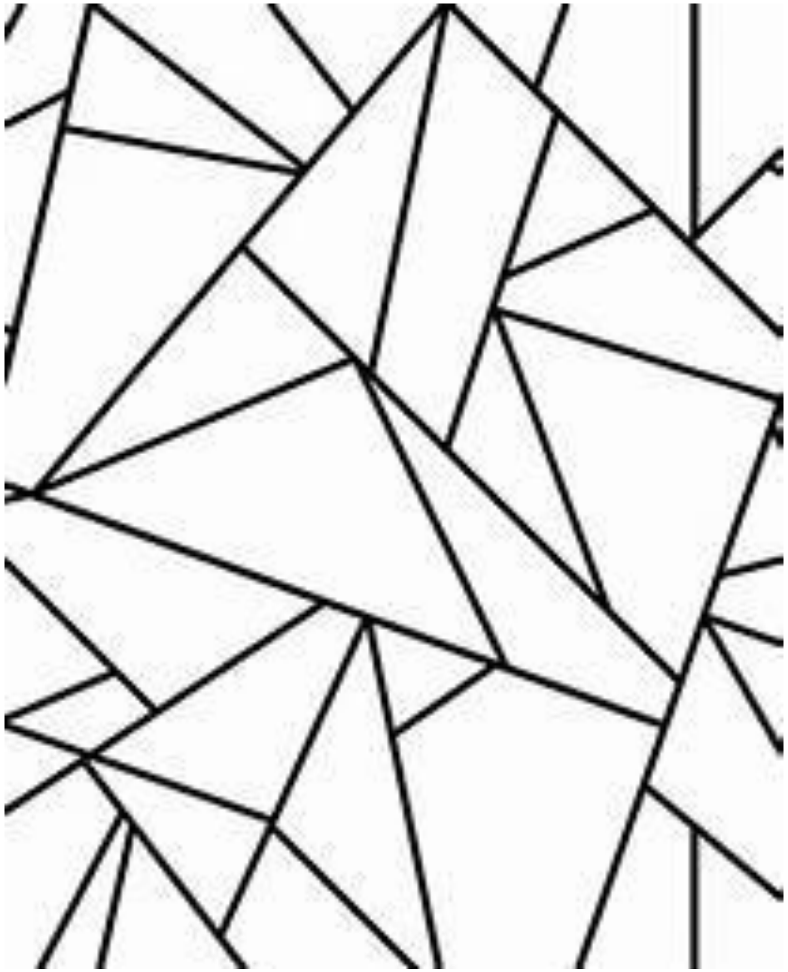
WORKING AT A*/A

(1) The diagram below shows a scatter graph. The data shows the number of months 5 students have had maths tutoring and the % they get in a test at the end of their tutoring.



(a) Draw a line of best fit on a graph similar to the one shown above.
(b) Find an equation for this line in the form $P = aM + b$ where P is their test % and M is the number of months they have been tutored for.
(c) Interpret, in context the constants a and b
(d) A student had had 2 months of tutoring. Use the model to predict the % they would get in their test.
(e) Explain 2 limitations of the model.
(f) Explain why the model doesn't show direct proportion.
30 more students enrolled in the tutoring. A model was found for all 30 students. The new model was $P = 12M + (b - 5)$
(g) What assumptions can you make about the new students who joined in comparison to the original 5 students

Circle Geometry



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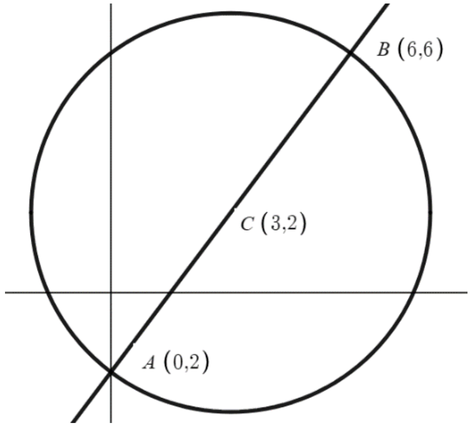
(30) Circle Geometry Midpoint & Perpendicular

WORKING AT D/E

(1) The line AB is a diameter of a circle. The coordinates of A and B are (6,3) and (12,5).
(a) Find the centre of the circle.
(b) Find the exact length of the diameter of the circle
(c) Hence, write down the exact length of the radius.

(2) The centre of a circle has coordinates $(3,11)$. Point A (2,6) and $B(p,q)$ both lie on the circle such that AB is a diameter of the circle. Show that the $p = 4$ and $q = 16$.

(3) The diagram below shows a circle with centre C and diameter AB. Find the equation of the line shown perpendicular to the diameter. Give your answer in the form $ax + by = c$



WORKING AT B/C

(1) A circle has centre C and diameter AB where the coordinates of A and B are $(-1,4)$ and $(9,4)$.

(a) Find the centre of the circle
(b) Hence, show that the perpendicular bisector of AB is a vertical line stating its equation.

(2) A circle with diameter 10 has centre $(-3,6)$. The points $A\ (-9,2)$ and $B\ (3,p)$ lie on the circle. Given that AB is the diameter of the circle, find the value of p.

(3) A circle has diameter PQ where the coordinates of P and Q are $(3,-4)$ and $(6,5)$ respectively.

Show that the diameter of the circle perpendicular to PQ has the equation $x + 3y - 6 = 0$

WORKING AT A*/A

(1) A circle lies in the xy plane and has centre (p,q). The coordinate axes are tangents to the circle. The points ABD and E lie on the circle. AB is a horizontal line and a diameter of the circle. DE is a vertical line and is also a diameter of the circle. Show that the area of the quadrilateral $AEBD = p^2 + q^2$

(2) The centre of the circle C, which lies in the xy plane, has coordinates (m,n). One diameter of the circle lies on the line with equation $x + y = 6$. Given that the coordinate axes are tangents to the circle, show that $m = n$

(3) A circle has centre $(0,0)$. The point $A(x,y)$ lies on the circle.
(a) Write down **any** other point that lines on the circle on the circle in terms of x and y.
(b) Find the exact length of the diameter of the circle in terms of x and y.

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(31) The Equation of a Circle

WORKING AT D/E

(1) Write down the equation of a circle with centre $(0,0)$ and radius 6.

(2) Find the equation of a circle with centre $(-3,6)$ and diameter 14.

(3) Find the centre and radius of the circle with equation $x^2 + y^2 + 2x - 4y - 20 = 0$

WORKING AT B/C

(1) A circle has equation
$(x-4)^2 + (y+p)^2 = 97$

Given that the point $(0,2)$ lies on the circle, find the two possible values of p.

(2) Show that the length of the radius of the circle with equation $x^2 + y^2 + 3x - 5y - 2 = 0$ is $\frac{\sqrt{42}}{2}$

(3) Explain why the circle with equation $(x-8)^2 + (y+10)^2 = 60$ doesn't cross any of the coordinate axes.

WORKING AT A*/A

(1) A circle in the xy plane has centre $(4,6)$ and radius $2\sqrt{5}$. Given that the point P with coordinates $(7, p)$ lies inside the circle, find the set of possible values of p.

(2) A circle with equation $x^2 + (y+8)^2 = r^2$ crosses the x axis at two points.
(a) Find the set of values for with r is valid
(b) Write down the equations of the horizontal tangents to the circle when $r^2 = 100$.

(3) A circle has equation
$x^2 + y^2 - 6x + 2py + 12 = 0$ where p is a constant. Find the set of possible values of p.

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(32) Circles and Straight Lines (Intersections)

WORKING AT D/E

(1) Show, using simultaneous equations, the line with equation $y = 3$ intersects the circle with centre (0,0) and radius 5 in two places giving the coordinates of the points of intersection.

(2) (a) Find where the circle with equation $(x-3)^2 + (y+2)^2 = 45$ crosses the y axis.

(b) Sketch the graph of $(x-3)^2 + (y+2)^2 = 45$ showing that the circle crosses the x axis at the points $(3+\sqrt{41}, 0)$ and $(3-\sqrt{41}, 0)$

(3) By drawing two graphs, show that the line $y = -x$ is not a tangent or a chord to the circle with equation $(x+5)^2 + (y+6)^2 = 1$

WORKING AT B/C

(1) Show, using algebra, that the line $y = x$ is a chord to the circle with equation $(x-4)^2 + (y-3)^2 = 1$ finding the points where the two graphs meet.

(2) A circle has equation

$$(x+3)^2 + (y-5)^2 = 25$$

(a) Write down the centre of the circle and the radius length.
(b) Show that the line with equation $3x - 4y + 29 = 0$ passes through the circle at two points, finding the points of intersection.
(c) Hence, show that the line creates the diameter of the circle.

(3) Prove, using the discriminant, that The line with equation $y = \frac{4x-31}{3}$ is a tangent to the circle with equation $(x-3)^2 + (y-2)^2 = 25$

WORKING AT A*/A

(1) The line with equation $y = mx$ is a tangent to the circle with equation $(x+2)^2 + (y+1)^2 = 1$. Show, using algebra, that either $m = 0$ or $m = \frac{3}{4}$

(2) Given that the line with equation $y = \frac{1}{2}x + c$ is a chord to the circle with equation $x^2 + (y-8)^2 = 20$, find the range of possible values of c.

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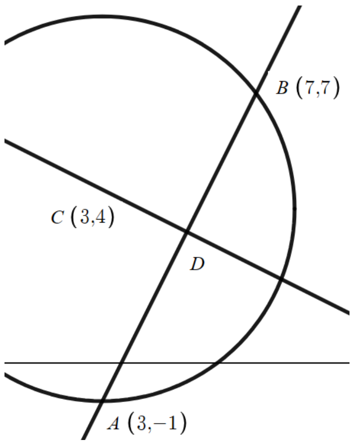
(33) Circles (Tangents and Chords)

WORKING AT D/E

(1) Find the equation of the tangent to the circle with equation $x^2 + y^2 = 100$ at the point $(6,8)$. Give your answer in the form $y = mx + c$ where m and c are simplified fractions.

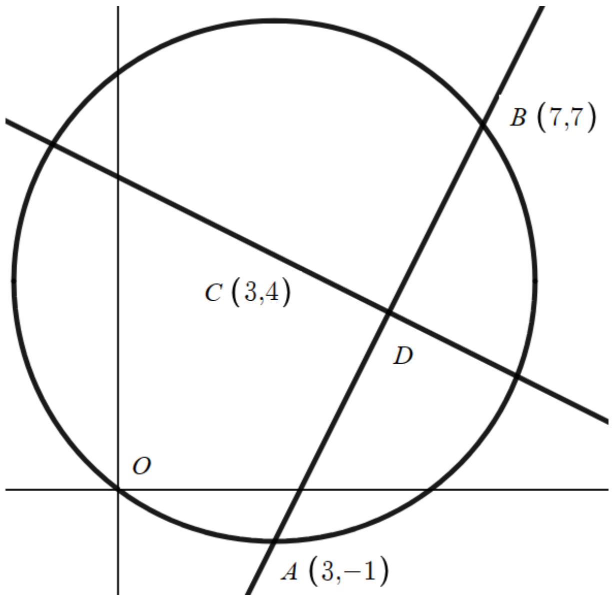
(2) The diagram below shows part of a circle. The line ADB is a chord to the circle and C is the centre of the circle. Given that the line segment CD is part of the radius, find the coordinate of D.



(3) A circle has centre C. A tangent is drawn to the circle at the point P. The gradient of the tangent at P is m. Write down the gradient of the radius CP giving your answer in terms of m.

WORKING AT B/C

(1) The diagram shows a circle centre C and chord ADB. The line CD lies on the radius of the circle.



(a) Find the equation of the circle.
(b) Show that the coordinates of D are $(5,3)$
(c) Hence, find the exact length of the line CD.

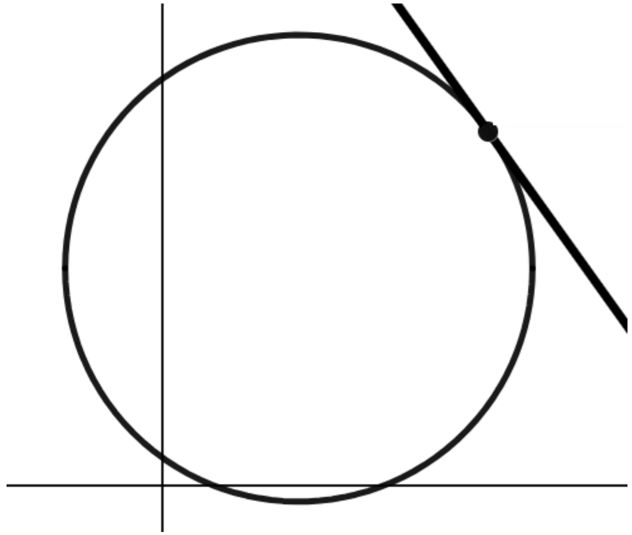
(2) Find the equation of the tangent to the circle with equation $(x-2)^2 + (y+7)^2 = 20$ at the point $(4,-3)$. Give your answer in the form $ax + by = c$.

(3) A circle has equation $x^2 + y^2 = 16$. Find the equation of any vertical or horizontal tangents to the circle.

WORKING AT A*/A

(1) Circle C has points $A\ (1,15)$, $B\ (6,14)$ and $C\ (-4,-10)$. By considering 2 different chords, prove that the centre of the circle C has coordinates $(1,2)$

(2) The diagram below shows a circle with equation $(x-a)^2 + (y-8)^2 = r^2$. The tangent to the circle at the point $(12,13)$ has gradient -1.4



Find the value of the constant a.

(3) A circle has centre $(0,0)$ and radius $5\sqrt{5}$. The tangents at the points A and B have a gradient of 2. Show that the coordinates of A and B have integer values.

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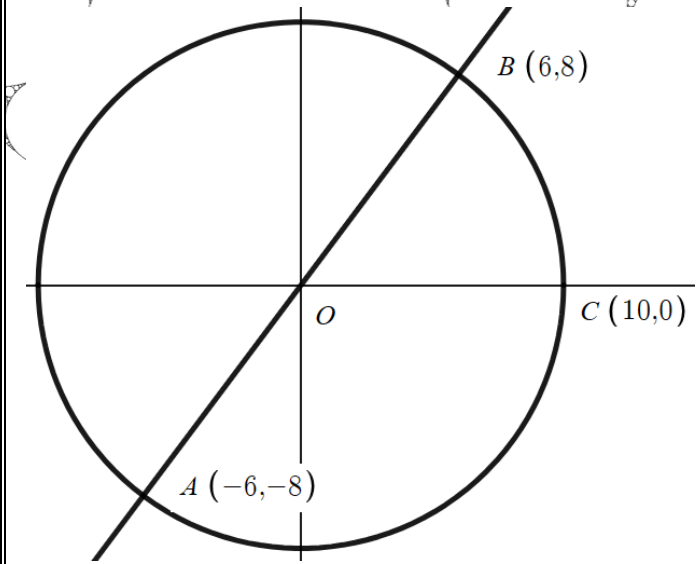
(34) Circles and Triangles

WORKING AT D/E

(1) The diagram below shows the circle with equation $x^2 + y^2 = 100$



(a) Verify that the point C $(10,0)$ lies on the circle.
(b) Write down the length of the radius of the circle.
(c) Prove that AB is a diameter of the circle.
(d) Find the size of the angle ACB in degrees.
(e) Given that O is the origin of the circle, find the area of the triangle OBC.
(f) The point D also lies on the circle. Given that the gradient of the chord AD is 0, find the coordinates of the point D.

WORKING AT B/C

(1) A circle when centre C has equation
$(x-3)^2 + (y-3)^2 = 10$
(a) Sketch the circle showing the coordinates of C.
The line with equation $y = 4$ cuts the circle at the points A and B.
(b) Find the coordinates of the points A and B.
(c) Find the area of the triangle ABC.

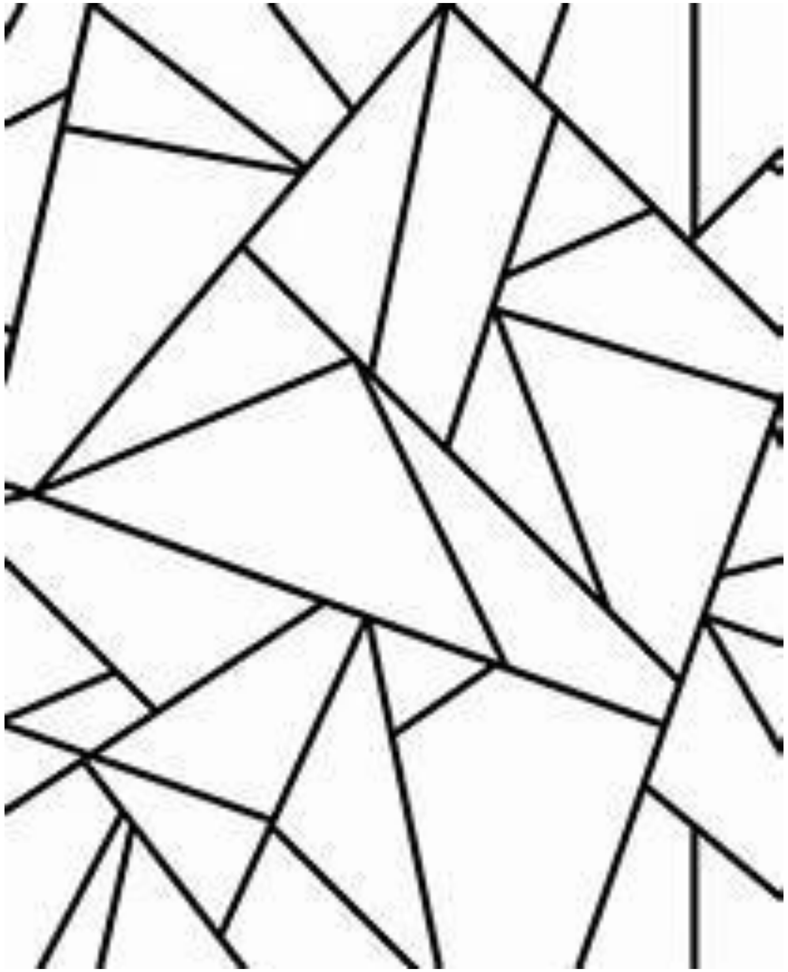
(2) A circle has equation $(x-6)^2 + (y+1)^2 = 29$
(a) Verify that the points P $(1,1)$ and Q $(4,4)$ both lie on the circle.
(b) Explain why PQ is not a diameter of the circle.
(c) The circle has centre C. Write down the coordinates of C.
(d) Hence, show that the perimeter of the triangle PCQ can be written in the form $a\sqrt{b} + c\sqrt{d}$ where a, b, c and d are integers.
(e) Show that the point $(7,5)$ lies outside the circle.

WORKING AT A*/A

(1) Points P $(4,1)$, Q $(9,6)$ and R $(6,7)$ lie on the circle C. Prove that PQ is a diameter of the circle.

(2) The line $x = 0$ is a tangent to the circle with equation $(x-4)^2 + (y-3)^2 = r^2$.
(a) Write down the value of r^2
The circle crosses the line $y = 0$ at A and B, where $B > A$
(b) Show that the chord AB has length $2\sqrt{7}$
Given that the centre of the circle is C find the area of the triangle ACB in the form $p\sqrt{q}$

More Algebra



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(35) Algebraic Fractions

WORKING AT D/E

(1) Fully simplify $\frac{12x^4+4x^3+8x}{2x}$

(2) Show that $\frac{x^2+4x+4}{x+2}$ can be written as $ax+b$ where a and b are integers to be found.

(3) Show that $\frac{x^2-x-12}{x-4}$ simplifies to $x+3$

WORKING AT B/C

(1) Fully simplify $\frac{6x^2+13x+2}{2x+4}$

(2) Show that $\frac{6x^2-6}{x^3-x}$ simplified to $\frac{A}{x}$ where A is an integer to be found.

(3) Alan simplifies the fraction $\frac{2x^2+x-15}{2x^2-13x+20}$ to $\frac{x+3}{x-4}$

(a) Is he correct? You give a reason.

(b) Beryl then suggests that he can simplify further by cancelling the x terms to give $\frac{3}{-4}$. Is Beryl right? You must give a reason for your answer.

WORKING AT A*/A

(1) Fully simplify $\frac{A^2x^4-B^2y^4}{Ax^2+By^2}$

(2) Fully simplify $\frac{144x^2-25x^4}{10x-24x^2}$

(3) Show that $\frac{(A+1)^{30}-(A+1)^{29}}{2A+2} \equiv \frac{A}{2}(A+1)^{28}$

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(36) Polynomial Division

WORKING AT D/E

(1) Show, using long division that there is no remainder when $x^3 + 4x^2 - 15x - 18$ is divided by $(x + 1)$

(2) (a) Show, using long division that there is no remainder when $x^3 + 13x^2 + 52x + 60$ is divided by $(x + 2)$
(b) Hence, show that $x^3 + 13x^2 + 52x + 60$ can be factorised to give $(x + 2)(x + a)(x + b)$ where a and b are integers.

(3) Using polynomial division, find the remainder when $x^3 + 3x^2 - 16x + 7$ is divided by $(x - 3)$

WORKING AT B/C

(1) (a) Show, using polynomial division that when $x^3 - 7x - 6$ is divided by $(x + 1)$ there is no remainder.

(b) Hence write $x^3 - 7x - 6$ in the form $(x + 1)(x + a)(x + b)$

(2) (a) Show, using polynomial division that when $2x^3 + 13x^2 - 8x - 7$ is divided by $(2x + 1)$ there is no remainder.

$$g(x) = 2x^3 + 13x^2 - 8x - 7$$

(b) Using your answer to part (a) show that the solutions to $g(x) = 0$ are $x = -\frac{1}{2},\ x = 1$ and $x = -7$.

(3) (a) Show, using polynomial division that $x^3 - 4x^2 - 2x - 15$ has no remainder when divided by $(x - 5)$

(b) Using your answer to part (a) show that $x = 5$ is the only real solution to the equation

$$x^3 - 4x^2 - 2x - 15 = 0$$

WORKING AT A*/A

(1) When $x^3 + 1$ is divided by $(x + 1)$ there is no remainder. Use polynomial division to express $x^3 + 1$ as a product of three linear factors.

(2) (a) The volume of a cuboid can be written as $V = x^3 + 2x^2 - 11x - 12$. One side length has an express of $x + 4$. Find an expression for the lengths of the remaining two sides in the form $(x + a)$ and $(x + b)$ where a and b are integers.

(b) State, with a reason why $x > 3$

(3) Show, using polynomial division that $x^2 + 1$ is a factor of $x^4 - 1$ and find the remaining factors of $x^4 - 1$.

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(37) The Factor and Remainder Theorem

WORKING AT D/E

(1) $f(x) = x^3 - 2x^2 - 13x - 10$
(a) Using the factor theorem, show that $(x + 1)$ is a factor of $f(x)$.
(b) Using the factor theorem, show that $(x - 2)$ is not a factor of $f(x)$.
(c) Given that $f(5) = 0$ and $f(-2) = 0$, fully factorise $f(x)$

(2) $g(x) = 2x^3 + x^2 + px + 12$
(a) Given that $(x - 3)$ is a factor of $g(x)$, show that $p = -25$
(b) Using long division, fully factorise $g(x)$
(c) Using your answer to part (b), solve $g(x) = 0$

(3) $h(x) = 3x^3 + bx^2 + cx + d$ where b, c and d are constants.
Given that $h(3) = 11$ and $h\left(-\frac{1}{3}\right) = -4$
(a) What statement can be made about the expression $(x - 3)$?
(b) What statement can be made about the expression $(3x + 1)$?

WORKING AT B/C

(1) $f(x) = x^3 - 2x^2 - 5x + 6$
(a) Use the factor theorem to find a linear factor of $f(x)$ in the form $(x + a)$. You must show full workings.
(b) Use polynomial division to express $f(x)$ in the form $f(x) = (x + a)(x + b)(x + c)$
(c) Hence, solve the equation $f(x) = 0$
(d) Draw the graph of $y = f(x)$ showing where the curve crosses the coordinate axes.

(2) $g(x) = 4x^3 + px^2 + qx - 12$
Given that $(x + 2)$ and $(4x + 1)$ are factors of $g(x)$, show, using the factor theorem, that:
(a) $p = -15$ and $q = -52$
(b) Hence, fully factorise $g(x)$ showing full workings. Calculator methods are not accepted.

WORKING AT A*/A

(1) $f(x) = ax^3 + bx^2 + cx - 2$ where a, b and c are constants.

Use the following 3 facts to solve the equation $f(x) = 0$

$$f(1) = 0$$

$$f\left(-\frac{2}{3}\right) = 0$$

When $f(x)$ is divided by $(x - 2)$ the remainder is 40

You must show full workings.

(2) $g(x) = x^4 + x^3 - 6x^2 + 6x - 72$
(a) Show that $g(3) = 0$
(b) Using your answer from part (a), express $g(x)$ in the form $g(x) = (Ax^3 + Bx^2 + Cx + D)(x + E)$
(c) Given further that $(x + 4)$ is the only other factor of $g(x)$, sketch the graph of $y = g(x)$ showing any points where the curve crosses the coordinate axes.

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(38) An Introduction to Mathematical Proof

WORKING AT D/E

(1) Prove that

$$(x-3)(2x+1)^2 \equiv 4x^3 - 8x^2 - 11x - 3$$

(2) $f(x) = x^2 + 2x + 6$

Prove that $f(x)$ is always positive for all real values of x

(3) Prove that $\frac{x}{2+\sqrt{3}} \equiv x\left(2-\sqrt{3}\right)$

WORKING AT B/C

(1) The triangle ABC has coordinates $A\ (6,8)$, $B\ (2,4)$ and $C\ (3,3)$

Prove that $< ABC$ is a right angle.

(2) $f(x) = x^2 + 4x + c$ where c is a constant.

Prove that the minimum point on the graph of $y = f(x)$ has coordinates $(-2, c-4)$

WORKING AT A*/A

(1) Prove that, if $y = 3x + c$ where c is a constant, is a chord to the circle with $x^2 + y^2 = 36$ then c must satisfy the inequality $-6\sqrt{10} < c < 6\sqrt{10}$.

(2) In the triangle ABC, $< ABC = x^o$

The coordinates of A, B and C are (a, b), (c, d) and (e, d) respectively.

Prove that if $x = 90$ then $a = c$.

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(39) Methods of Proof

WORKING AT D/E

(1) Prove that the difference between any two prime numbers is not always an even number.

(2) If n is a single digit odd number, prove that $n+1$ is not always a single digit even number.

(3) Prove, by counter example, that $2n^2+1$ for all positive integers n is not always a prime number.

WORKING AT B/C

(1) (a) Given that $2n$ is always even, show that the sum of the squares of two consecutive even numbers can be written as

$$8n^2+8n+4$$

(b) Hence, prove that the sum of the squares of two consecutive even numbers is always divisible by 4.

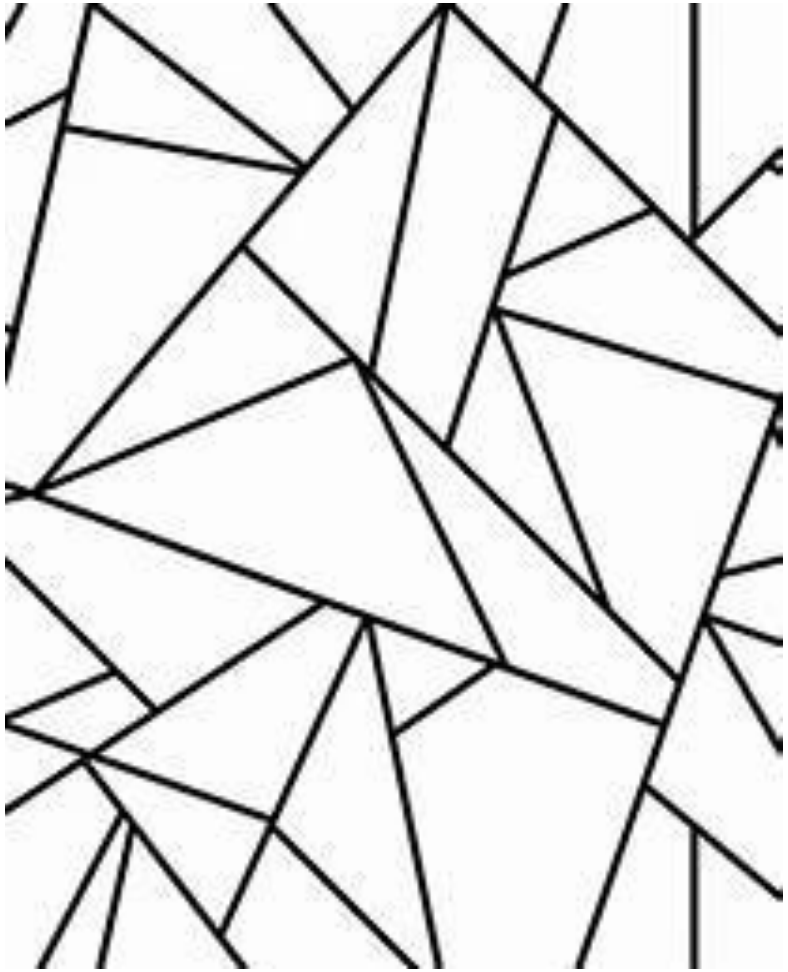
(2) Prove that the difference between the cubes of any consecutive integers is always one more than a multiple of 3.

WORKING AT A*/A

(1) Prove, that if a and b are both positive numbers, then $\frac{a^2+b^2}{2ab} \geq 1$

(2) Prove, that if x and y are both positive integers, then $\frac{y}{x}+\frac{x}{y} \geq 2$

Binomial Expansions



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(40) Binomial Expansion (Using Pascal's Triangle)

WORKING AT D/E

(1) Use Pascal's Triangle to expand $(2+x)^4$ in ascending powers of x.

(2) Using Pascal's Triangle, find the term in x^3 in the expansion of $(1-2x)^5$

(3) Show that the term in x in the expansion of $(1+x)(2+x)^4$ is $48x$

WORKING AT B/C

(1) Using Pascal's Triangle, show that the coefficient of the term in x^2 in the expansion of $(3-2x)^5$ is 1080

(2) Use Pascal's Triangle to show that there is no term in x^2 in the expansion of $(1+x)(1-x)^3$

(3) In the expansion of $(a+2x)^4$ where a is a constant, the term in x^2 is 216.

Show, using the binomial expansion, find the possible values of a.

WORKING AT A*/A

(1) In the expansion of $(1+x^{-1})(1+x)^4$, show that the coefficient of the term in x^2 is 10.

(2) In the expansion of $(p+qx)^4$ where p and q are positive constants, the term independent of x is 81 and the term in x^3 is $\frac{4}{9}$.

Find the values of p and q.

(3) (a) Show that the expansion of $(1+\sqrt{2})^4$ can be written in the form $a+b\sqrt{2}$

(b) Without any further expansions, explain why

$$(1+\sqrt{2})^4+(1-\sqrt{2})^4=2a$$

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(41) Binomial Expansion (Factorial Notation)

WORKING AT D/E

(1) Without a calculator, find the value of 5!

(2) Without a calculator, show that $\binom{5}{3} = 10$

(3) Given that $\binom{4}{r} = 4$, find the possible values of r.

WORKING AT B/C

(1) Given that $\binom{18}{m} = \frac{18!}{3!}$ write down the possible values of m.

(2) Simplify $n(n-1)!$

(3) Given that $\binom{16}{5} = 4368$, write down the value of n such that $\binom{16}{n} = 4368$, $n \neq 5$.

WORKING AT A*/A

(1) Show, with full workings, that $\binom{n}{1} = n$

(2) Show, with full workings, that

$$\binom{n}{3} = \frac{n^3 - 3n^2 + 2n}{6}$$

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(42) Binomial Expansion (The $\binom{n}{r}$ Method)

WORKING AT D/E

(1) Use the binomial expansion to find the full expansion of $(2+3x)^4$ in ascending powers of x.

(2) Use the binomial expansion to show that the first 4 terms in the expansion of $\left(1+\frac{1}{4}x\right)^8$ are

$$1+2x+\frac{7}{4}x^2+\frac{7}{8}x^3$$

You must show full workings.

(3) Find the full expansion of $(1-x)^5$ simplifying each term.

WORKING AT B/C

(1) (a) Find the full expansion of $(a+b)^5$

(b) Hence, write down the expansion of $(a-b)^5$

(2) Find the full expansion of $\left(2+\frac{x}{2}\right)^4$ in ascending powers of x. Write each coefficient in their simplest form.

(3) Show that the term in x^7 in the expansion of $\left(5-\frac{x}{3}\right)^{11}$ is $-\frac{68750}{729}x^7$

WORKING AT A*/A

(1) Show that

$$\left(a+\frac{1}{a}\right)^4+\left(a-\frac{1}{a}\right)^4\equiv\frac{2}{a^4}(a^8+6a^4+1)$$

(2) (a) What is the maximum possible number of terms in the expansion of $(a+b)^n$ where n is a positive integer? Give your answer in terms of n.

(b) Write an expression for the seventh term in the expansion $(a+b)^n$ in terms of a, b and n.

(3) Alan claims that when n is an even positive integer in the expansion of $(x+x^{-1})^n$ there will always be a term independent of x. Is he correct? You must justify your answer.

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(43) Binomial Expansion (Problem Solving)

WORKING AT D/E

(1) (a) Show that the expansion of $(1+2ax)^4$ can be written as

$$1+8ax+24a^2x^2+32a^3x^3+16a^4x^4$$

(b) Given that the term in $x = 24$, find the value of a

(c) Hence, find the coefficient of the term in x^2

(2) (a) Show that the first 3 terms of the expansion of $(1+x)^7$ are $1+7x+21x^2$

(b) Hence, show that the first 3 terms in the expansion of $(1-x)(1+x)^7$ are $1+6x+14x^2$

(3) In the expansion of $(2+px)^6$ the coefficient of the term in x is 960.
Show, using the binomial expansion, that $p=5$

WORKING AT B/C

(1) (a) Find the first 3 terms of the expansion $(p+3x)^6$, where p is a positive constant. Give your answer in ascending powers of x fully simplifying each term.

(b) Given that the coefficient of the term in x is **twice** that of the term in x^2, show that

$$p^4(p-15)=0$$

(c) Hence, write down the value of p.

(d) Find the coefficient of the term in x.

(2) (a) Use the binomial expansion to find the full expansion of $(1+x)^5$ in ascending powers of x.

(b) Using your answer to part (a), write down the first 3 terms in the expansion of $(1-2y)^5$

WORKING AT A*/A

(1) (a) Find the terms up to an including the term in x^3 in the expansion of $(3+x)(1+px)^7$ where p is a negative constant. Give each term in its simplest form.

(b) Given that the coefficient of the term in x^2 is 238, find the coeffieint of the term in x^3

(2) In the expansion of $(p-x)(1+2x)^8$ where p is a constant. The first 2 terms in ascending powers of x are $A+Bx^2$ where A and B are constants.

Find the values of A, B and p.

(3) In the expansion of $(p+x)(q+x^3)^n$ where n, p and q are positive constants, the highest power of x is x^{19}. How many terms are there in the expansion of $(p+x)(q+x^3)^n$?

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(44) Binomial Expansion (Estimations and Approximations)

WORKING AT D/E

(1) (a) Find the terms up to an and including the term in x^2 in the expansion of $(1+x)^7$

(b) By choosing a suitable value of x, use your answer to part (a) show that a quadratic approximation to 1.01^7 is 1.0721

(2) (a) Find the first 4 terms in the expansion of $(1-2x)^{12}$ is ascending powers of x.

(b) Use your answer to part (a) to find an approximation to the expansion of 0.96^{12}

WORKING AT B/C

(1) (a) Find the first 3 terms in the expansion of $(2-\frac{x}{4})^8$ in ascending powers of x, simplifying each term.

(b) Using your answer to part (a), find a quadratic approximation for 1.99^8

(c) Show that the percentage error for the approximation is less than 1%.

(2) (a) Find the first 3 terms in the expansion of $\left(5-\frac{x}{3}\right)^9$ in ascending powers of x. Simplify each coefficient fully.

(b) If x is small and terms in x^2 and higher can be ignored, show that

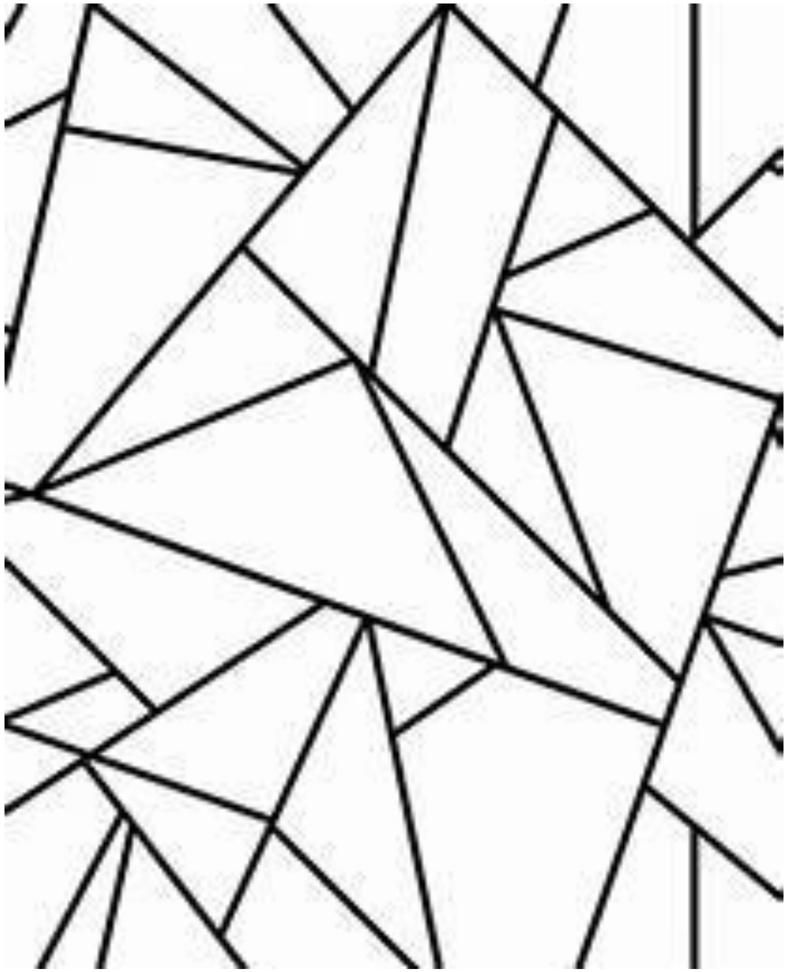
$\left(\frac{1}{5}+x\right)\left(5-\frac{x}{3}\right)^9 \approx 390625+1718750x$

WORKING AT A*/A

(1) If x is small and terms in x^2 and higher can be ignored, show that $(a+x)^n(a-x)^n \approx a^{2n}$ when a and n are positive integers.

(2) Use the binomial expansion of $(5-4x)^8$ to find a cubic approximation for 4.92^8 giving your answer to 1 decimal place.

Trigonometry (Triangles)



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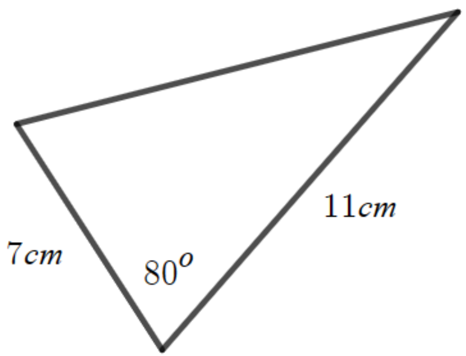
(45) The Cosine Rule

WORKING AT D/E

(1) A triangle has side lengths $4cm$, $5cm$ and $6cm$.
(a) Prove that the triangle is not a right-angled triangle.
(b) Use the cosine rule to find the size of the smallest angle in the triangle to 3 S.F.

(2) Show that the perimeter of the triangle below is $30.0cm$ correct to 3 significant figures.

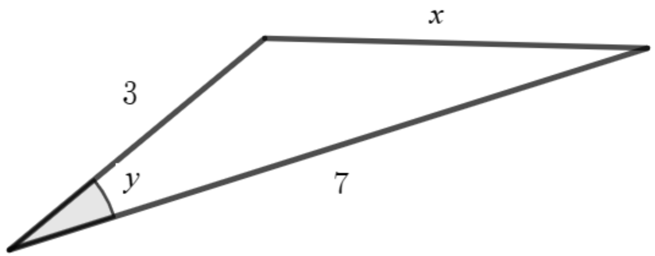


(3) In a right-angled triangle $AB = 2, BC = 3$ and $AC = \sqrt{13}$. One angle in the triangle has size x. Find the smallest possible value for of $\cos(x)$. Give your answer in exact form.

WORKING AT B/C

(1) Beryl walks from home on a bearing of 070^0 for $6km$ before stopping. She then walks on a bearing of 112^0 for $11km$ before stopping.
(a) Find how far from home Beryl now is giving your answer to 3 S.F.
(b) Find the bearing she is now on from home giving your answer to 3 S.F.

(2) The diagram below shows a triangle with side lengths $3, 7$ and x and an angle with size y.

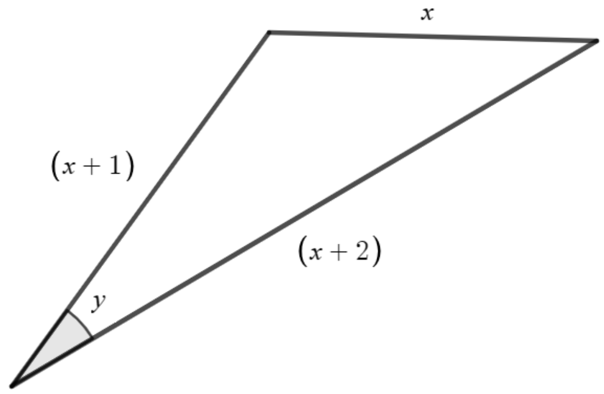


The diagram is not drawn to scale.
(a) Show that $\cos(y) = \frac{58-x^2}{42}$
(b) Given further than $y = 30^o$ show, without a calculator, that $x = \sqrt{58 - 21\sqrt{3}}$

(3) A triangle has side lengths in the ratio 2:3:4. Show that the value of the cosine of the largest angle in the triangle will be $\frac{-1}{4}$.

WORKING AT A*/A

(1) The diagram below shows a triangle with side lengths $x, (x+1)$ and $(x+2)$ and an angle with size y.



The diagram is not to scale.

(a) Show that $\cos(y) = \frac{x^2+2x+3}{2x(x+2)}$

(b) In a different triangle $\cos(w) = \frac{x^2+x+1}{2x+3}$. Show that the angle w cannot be a right angle.

(2) Prove, using the cosine rule, that if an isosceles triangle has one side length 1 unit longer than the other two, the angle between the shorter sides will only be obtuse if the longest side is less than $2 + \sqrt{2}$ units.

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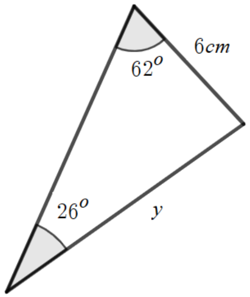
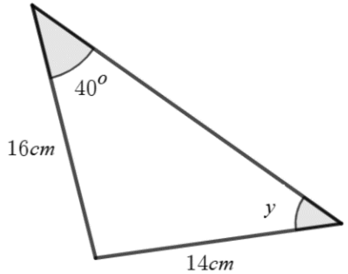
(46) The Sine Rule

WORKING AT D/E

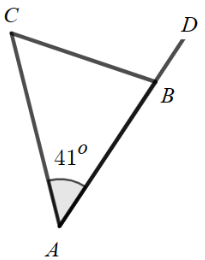
(1) Find the size of y in the diagram below. Give your answer to 3 significant figures.



(2) Find the size of angle y in the diagram below. Give your answer to 3 significant figures.



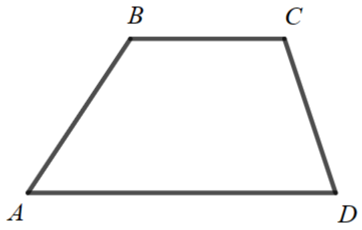
(3) In the diagram below $AB = 13cm$ and $CB = 12cm$. Find the size of $< CBD$ to 1 decimal place.



WORKING AT B/C

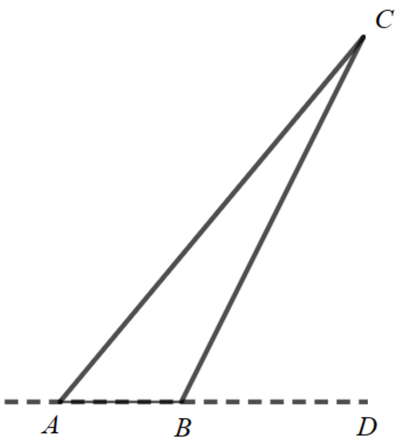
(1) In triangle $PQR, PQ = 12, QR = 11$ and $< QPR = 50^0$. Find the minimum possible length of PR giving your answer to 3SF.

(2) The diagram below shows the trapezium $ABCD$.



$BC = 11cm, CD = 15cm$ and $< BCD = 98^0$. Find the size of $< BDA$ giving your answer to 3SF.

(3) The diagram below shows ΔABC and the horizontal line ABD.



Given that $AB = 4.2, < CAB = 40^o$ and $< CBD = 55^o$, find the perpendicular height of the ΔABC relative to the line ABD.

WORKING AT A*/A

(1) In triangle $PQR, PQ = 2p, QR = q$ and $< QPR = 30^0$. Show that if $< QRP$ is obtuse, then $< QRP = 180 - \arcsin\left(\frac{p}{q}\right)$

(2) Alan walks from home on a bearing of 136^o for 7 miles before stopping for a rest. He then walks x miles on a bearing of 040^o before stopping.
(a) Given that he is now on a bearing of 098^o from his home, find the value of x to 2 decimal places.
Alan now walks home.
(b) Find the shortest possible length from his current position to his home.

(3) In the isosceles triangle PQR, $\sin(PQR) = 0.25$, $PQ = p$ and $PR = r$. Given that $p > r$, without a calculator show that the perimeter of the triangle can be written as

$$\frac{p}{4}\left(8 + \frac{1}{\sin(< QPR)}\right)$$

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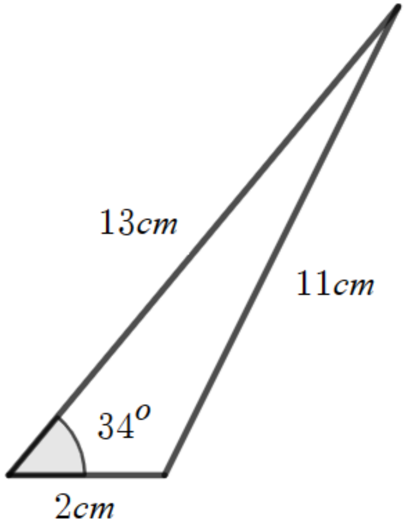
(47) Area of a Triangles

WORKING AT D/E

(1) Find the area of the triangle to 1 decimal place.



(2) A triangle has side lengths of 6cm, 7cm and 8cm.
(a) Find the size of any angle in the triangle
(b) Hence find the area of the triangle to 3SF.

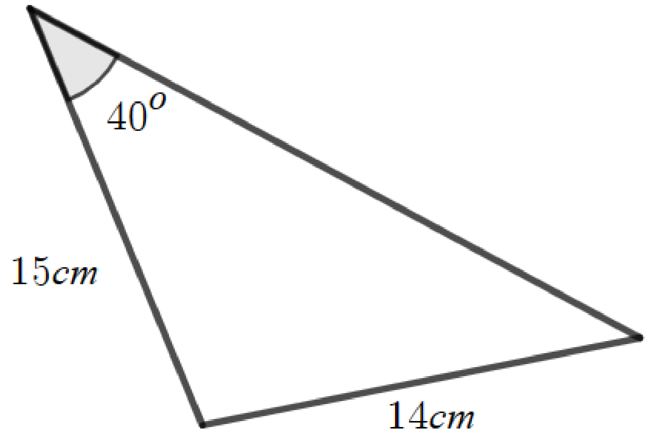
(3) An isosceles triangle has two side lengths of 7cm and two angles of 40^o. Show that the area of the triangle is 24.1cm² to one decimal place.

WORKING AT B/C

(1) In $\Delta ABC, AB = 4, BC = 3$ and $\sin(< ABC) = \frac{1}{8}$

Without a calculator, show that the area of $\Delta ABC = \frac{3}{4}$

(2) Find the area of the triangle shown giving your answer to 3 significant figures.



(3) Beryl is fencing off a piece of land from her home. She walks 280m from her home on a bearing of 058^0 and fences a straight line off. She then stops. She walks directly north for 132m fencing along a straight line. To complete the fenced off piece of land she walks directly home on a straight line and fences along the straight line.
Find the total area of the fenced off piece of land giving your answer to 1 decimal place.

WORKING AT A*/A

(1) In the triangle $ABC, AB = 2x, BC = (3x - 1)$ and $\sin(ABC) = 0.4$ where $< ABC$ is acute.

(a) Given that the area of the triangle is 0.8 units, show, without a calculator that $(3x + 2)(x - 1) = 0$

(b) Explain why the triangle is isosceles.

(2) An equilateral triangle has area $3\sqrt{3}$. Without a calculator show that the perimeter can be written in the form $a\sqrt{b}$ where a and b are integers to be found.

(3) In $\Delta ABC, AB = (1 + \sqrt{2}), BC = (1 + 2\sqrt{2})$ and $< ABC = \theta$. Given that the area of the triangle is $\frac{1}{2}$, show without a calculator, that $\sin(\theta)$ can be written as $\frac{5-3\sqrt{2}}{7}$.

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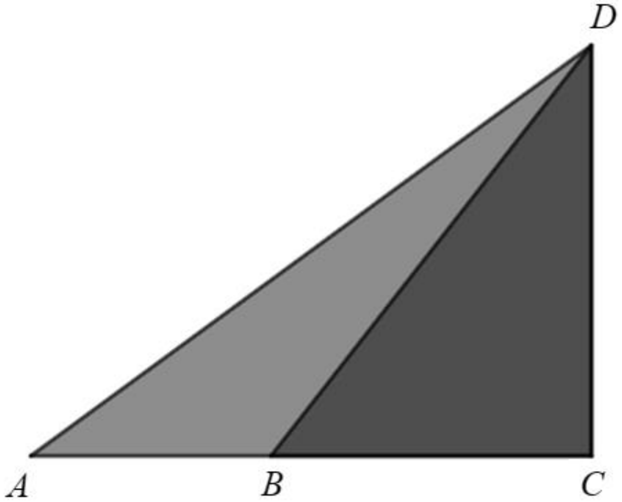
(48) Triangles (Problem Solving)

WORKING AT D/E

(1) The diagram below shows the right-angled triangle ACD where $< ACD = 90^0$. ABC and BD are both straight lines.

$AC = 7cm, BD = \sqrt{29}$ and $CD = 5cm$



(a) Find the size of $< ADB$.

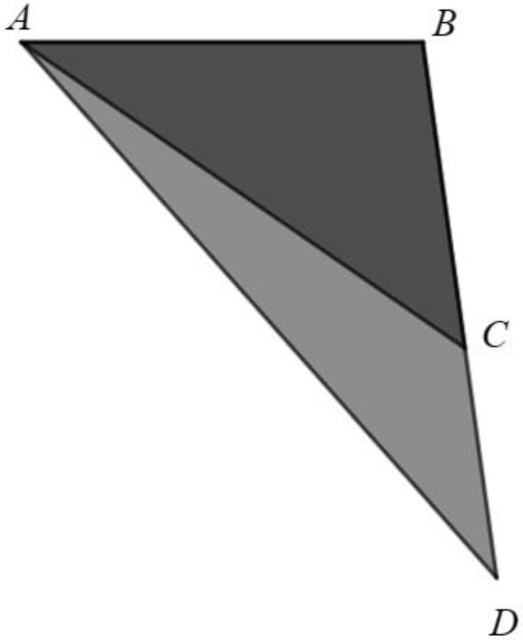
(b) Find the area of ΔADB.

(c) Find the perimeter of ΔADB to 3 SF.

WORKING AT B/C

(1) The diagram below shows the triangle ABD. BCD and AC are both straight lines.

$AB = 10cm$, $BD = 17cm, < DAC = 28^o$ and $< ABD = 93^0$



(a) Find AC to 1 decimal place.

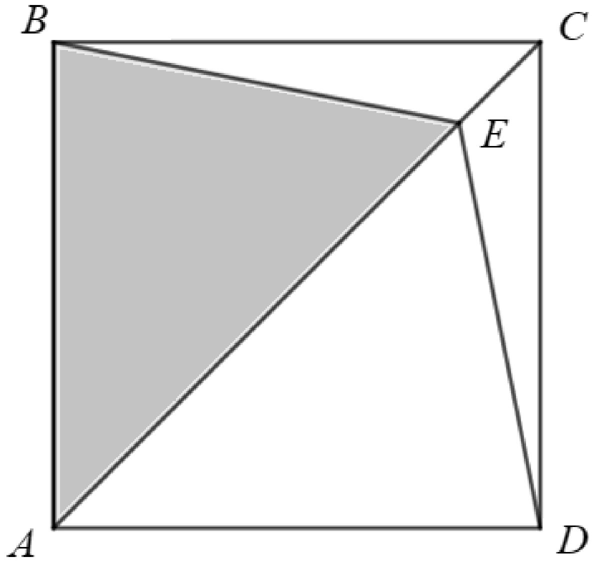
(b) Find what proportion of ΔABD is shaded darker grey?

(c) Find the perimeter of ΔABC

WORKING AT A*/A

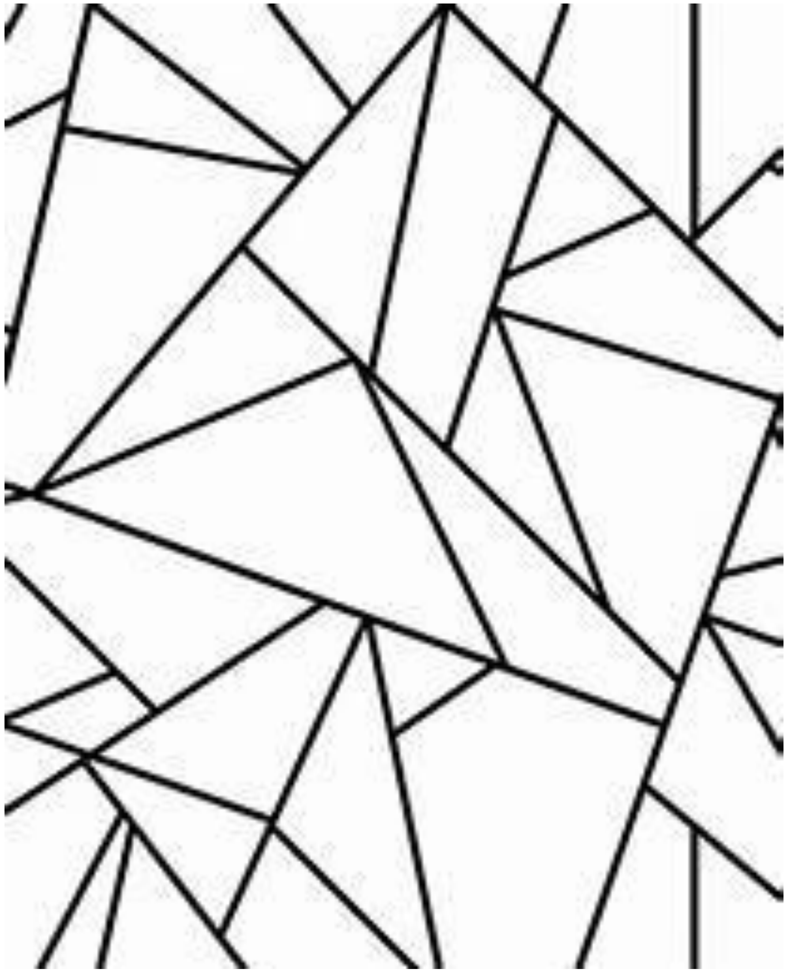
(1) A parallelogram has side lengths x and $2x$ and one interior angle θ. Given that the area of the parallelogram is 6 units, find the possible set of values of x.

(2) The diagram below shows a square $ABCD$ of area 36. AEC is a straight line and $CE = \sqrt{2}$.



Find the proportion of the square that is shaded giving your answer as a simplified fraction.

Trigonometry (Equations/Identities)



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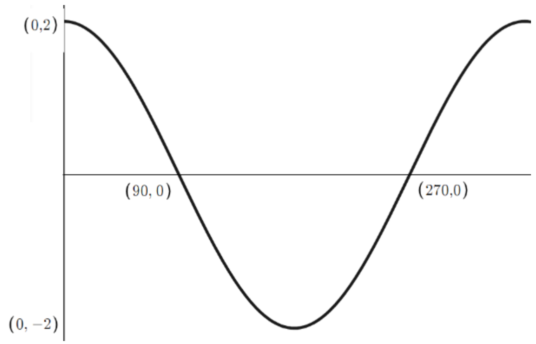
(49) Sine, Cosine & Tangent Graphs

WORKING AT D/E

(1) On separate sets of axes, sketch the graphs of:
(i) $y = \cos(x)\,, 0 \leq x \leq 360$
(ii) $y = \sin(x)\,, 0 \leq x \leq 360$
(iii) $y = \tan(x)\,, 0 \leq x \leq 360$
For each graph, show where the curve meets the coordinate axes, any maximum or minimum points and the equations of any asymptotes.

(2) Complete the following sentences:
The graph of $y = \sin(x)$ cycles every__________
The graph of $y = \tan(x)$ cycles every__________
The graph of $y = \cos(x)$ cycles every__________

(3) Beryl draws what she thinks is the graph of
$y = \sin(x)\,, 0 \leq x \leq 360$



Write down two errors with she has made.

WORKING AT B/C

(1) On separate sets of axes, sketch the graphs of:
(i) $y = \cos(x)\,, -360 \leq x \leq 360$
(ii) $y = \sin(x)\,, -360 \leq x \leq 360$
(iii) $y = \tan(x)\,, -360 \leq x \leq 360$
For each graph, show where the curve meets the coordinate axes, any maximum or minimum points and the equations of any asymptotes.

(2) (a) On the graph of $y = \sin(x)\,, 0 \leq x \leq 360$,
$y = \frac{\sqrt{3}}{2}$ when $x = 60^o$. Where else on the graph will
$y = \frac{\sqrt{3}}{2}$?
(b) On the graph of $y = \cos(x), 0 \leq x \leq 360$,
$y = 0$ when $x = 90^o$. Where else on the graph will
$y = 0$?
(c) On the graph of $y = \tan(x)\,, 0 \leq x \leq 540$,
$y = 1$ when $x = 45^0$. Where else on the graph will
$y = 1$?

(3) Write down any lines of symmetry for each graph
(i) $y = \cos(x)\,, -360 < x < 360$
(ii) $y = \sin(x)\,, -180 < x < 180$
(iii) $y = \tan(x)\,, -360 < x < 360$

WORKING AT A*/A

(1) By considering the graphs of sin, cos and tan, tick any of the following statements that are true:

1. $\sin(x) \equiv \sin(180 - x)$
2. $\sin(x) \equiv \sin(180 + x)$
3. $\cos(x) = \cos(360 + x)$
4. $\sin(x) \equiv \sin(360 + x)$
5. $\tan(x) = \tan(360 + x)$
6. $\cos(360 - x) = \cos(x)$
7. $\tan(x) = \tan(180 + x)$
8. $\sin(-x) \equiv -\sin(x)$
9. $\cos(x) \equiv \sin(90 - x)$
10. $\tan(-x) = -\tan(x)$
11. $\cos(x) \equiv \cos(180 + x)$
12. $\cos(x) = \cos(-x)$

(2) Find all of values of x given $0 < x < 360$ for
(a) $\tan(x) = -1$
(b) $\sin(x) = -\frac{\sqrt{3}}{2}$
(c) $\cos(x) = \frac{1}{\sqrt{2}}$
(d) $\cos(x) = -0.1$
(e) $\tan(x)$ is undefined.
(f) $\sin(x) = -\frac{\sqrt{5}}{2}$

(3) How many points of intersection are there between each pair of graphs for $0 \leq x \leq 360$?
(a) $y = \cos(x)$ and $y = \sin(x)$
(b) $y = \cos(x)$ and $y = \tan(x)$
(c) $y = \tan(x)$ and $y = \sin(x)$

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(50) Transforming Graphs (Trigonometry)

WORKING AT D/E

(1) On separate sets of axes, draw each graph for $0 \leq x \leq 360$ showing where the graph meets or crosses the coordinate axes. On your graph include the coordinates of any maximum or minimum points and the equations of any asymptotes.
(a) $y = 2\sin(x)$
(b) $y = \cos(x) + 1$
(c) $y = -\tan(x)$
(d) $y = \sin(x - 30)$
(e) $y = 3\cos(x)$
(f) $y = \cos(x + 60)$
(g) $y = -\cos(x)$
(h) $y = \sin(2x)$
(i) $y = \cos(0.5x)$
(j) $y = 2 + \sin(x)$
(k) $y = \tan(-x)$
(l) $y = 1 - \cos(x)$

(2) The graph of $y = \cos(x) + k$, where k is a positive constant, doesn't meet the x axis. Explain why $k > 1$.

WORKING AT B/C

(1) The graph of $y = k\cos(x)$ has a maximum point with coordinates $(360, \sqrt{2})$
(a) Find the value of k
(b) Find the coordinates of the first minimum point on the graph for $x > 0$

(2) The graph of $y = \tan(x - a)$ where a is a positive constant has an asymptote when $x = 120^0$
(a) Explain why a could be 30^0
(b) Give any other possible value of a

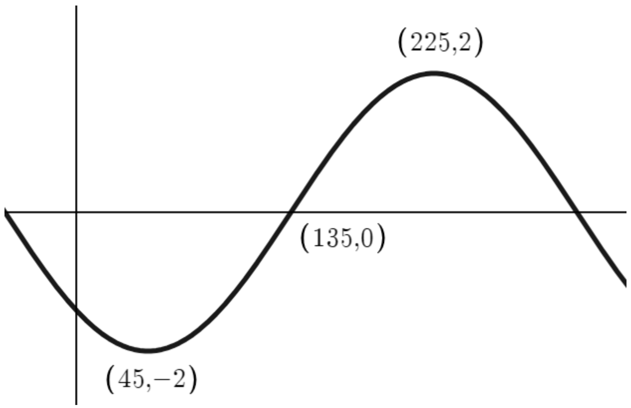
(3) Sketch the graph of $y = \sin(x) + a$, for $a > 1$ in the interval $0 \leq x \leq 360$.
Show the coordinates of the minimum and maximum point and where the graph crosses the y axis giving your answers in terms of a

WORKING AT A*/A

(1) (a) The graph of $y = \sin(ax)$, where a is a positive constant, meets the x axis in 7 places in the interval $0 \leq x \leq 360$. Find the value of a.

(b) The graph of $y = \sin(bx)$, where b is a positive constant, doesn't meet the x axis in the interval $0 < x \leq 360$. Find the possible set of values for the constant b.

(2) The diagram below shows the part of the graph of $y = a\cos(x + b)$ where a and b are constants.



Find possible values for a and b:
(a) If a is positive and b is negative
(b) If a is negative and b is negative
(c) If a is positive and b is positive
(d) If a is negative and b is positive

(3) Alan says that the graph of $y = tan(kx)$ where k is a positive constant has a single asymptote in the interval $0 \leq x \leq 90^o$. Find the set of values of k that would satisfy this statement.

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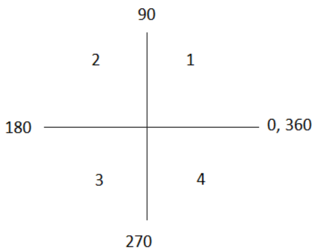
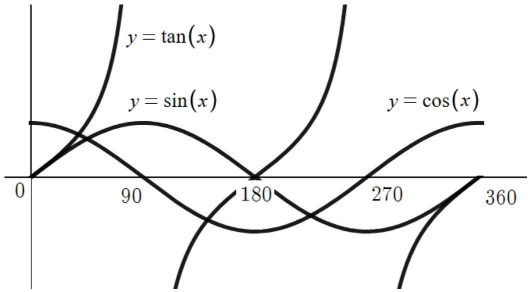
(51) The ‘CAST’ Diagram for Trig Ratios

WORKING AT D/E

(1) The diagram below shows the ‘4 quadrants’ from 0 to 360 degrees moving anticlockwise from 0. This is **<u>sometimes</u>** called the CAST diagram.



The diagrams below show the graphs of $y = \sin(x)$, $y = \cos(x)$ and $y = \tan(x)$ **on the same set of axes.**



(a) Label each graph above.

(b) Using the graphs, **<u>or otherwise</u>**, write where each trigonometric function is positive on the diagram above with the 4 quadrants.

(2) The value of $\cos(a) = -0.1$. Which two quadrants could it be in?

WORKING AT B/C

(1) For the following statements, write down the 2 quadrants the value will lie in. DO NOT CALCULATE THE ANGLE. The first one is done for you.

(a) $\sin(x) = -0.25$. This is the 3rd and 4th quadrant.

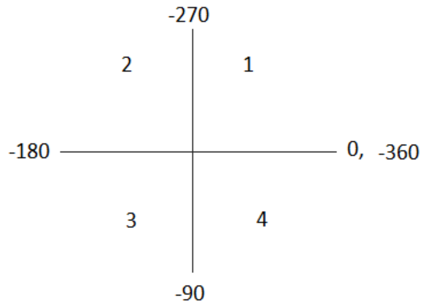
(b) $\cos(x) = 0.4$

(c) $\tan(x) = 3$

(d) $\cos(x) = -\frac{1}{5}$

(e) $\sin(x) = 0.63$

(2) You can also use the 4 quadrants for negative values by reading clockwise from 0.



Using the diagram above **<u>or otherwise</u>**, write down whether the following values will be positive or negative. DO NOT USE A CALCULATOR TO WORK OUT THEIR VALUE.

(a) $\sin(-80^o)$

(b) $\cos(-28^o)$

(c) $\tan(-100^o)$

(d) $\sin(-320^o)$

(3) Given that both $\sin(a)$ and $\cos(b)$ are negative, write down which quadrant they will be in.

WORKING AT A*/A

(1) Express each of the following in terms of either $\sin(x)$, $\cos(x)$ or $\tan(x)$.

(a) $\sin(-x)$

(b) $\cos(-x)$

(c) $\tan(-x)$

(d) $\sin(-180 + x)$

(e) $\cos(-360 + x)$

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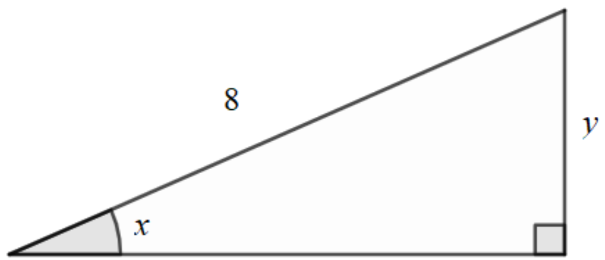
(52) Trigonometry (Exact Values)

WORKING AT D/E

(1) Without a calculator, find the exact vale of each:
(a) $\tan(60^o)$
(b) $\sin(45^o)$
(c) $\cos(30^o)$
(d) $\sin(60^o)$
(e) $\cos(90^o)$
(f) $\tan(30^o)$
(g) $\sin(360^o)$
(h) $\cos(180^o)$
(i) $\tan(90^o)$
(j) $\cos(45^o)$

(2) Given that $\sin(x) = 0.4$ show that the $y = 3.2$



(3) $\sin(a) = \cos(a)$ where a is a positive acute angle. Write down the value of a.

WORKING AT B/C

(1) Without a calculator, find the exact vale of each:
(a) $\tan(-30^o)$
(b) $\sin(225^o)$
(c) $\cos(-60^o)$
(d) $\sin(-60^o)$
(e) $\cos(135^o)$
(f) $\tan(210^o)$
(g) $\sin(-90^o)$
(h) $\cos(210^o)$
(i) $\tan(330^o)$
(j) $\cos(300^o)$

(2) $\sin(b) = \cos(b)$ where b is a positive obtuse angle. Write down the value of b.

(3) Without a calculator, show that

$$\tan(60) + 3\tan(-30) = 0$$

WORKING AT A*/A

(1) In the interval $0 \leq x \leq 360$, how many times will the $\sin(2x) = a$ where a is a constant and $0 < a < -1$?

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(53) Proving Trigonometric Identities

WORKING AT D/E

(1) Write down the two trigonometric identities that you will need to use in this unit.

(2) Simplify each of the following using your answers from question (1) to help you.

(a) $\frac{\sin(6x)}{\cos(6x)}$

(b) $\sqrt{1 - cos^2(x)}$

(c) $\sqrt{1 - sin^2(3x)}$

(d) $1 - sin^2(x)$

(e) $sin^2(8x) + cos^2(8x)$

(f) $6sin^2(\theta) + 6cos^2(\theta)$

(g) $\frac{sin^2(x)}{cos^2(x)}$

(h) $\frac{\sqrt{1-cos^2(4\theta)}}{\sqrt{1-sin^2(4\theta)}}$

(i) $\tan(x)\cos(x)$

(3) Show that

$$(\sin(x) + \cos(x))^2 \equiv 2\sin(x)\cos(x) + 1$$

WORKING AT B/C

(1) Using the identity

$$a^4 - b^4 \equiv (a^2 - b^2)(a^2 - b^2)$$

Show that

$$cos^4x - sin^4x \equiv cos^2x - sin^2x$$

(2) Prove each identity:

(a) $\frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta} \equiv \frac{1}{sin\theta cos\theta}$

(b) $\frac{3sin(A)}{\tan(A)} \equiv 3\cos(A)$

(3) (a) Given that $9sinx = 14cosx$, write down the value of $tanx$

(b) Given that $0 < A < 90$ and $\sin(A) = \frac{3}{5}$

(i) Show that $\cos(A) = \frac{4}{5}$

(ii) Find the value of $\tan(A)$

WORKING AT A*/A

(1) (a) Given that $180 < A < 270$ and $\sin(A) = -0.8$

(i) Find the exact value of $\cos(A)$

(ii) Find the exact value of $\tan(A)$

(b) How would your answer(s) change if $270 < A < 360$?

(2) (a) Given that $x = 4cos\theta$ and $y = 2 + 4sin\theta$, show that $x^2 + (y-2)^2 = k$ where k is a constant to be found.

(b) Given that $p = 1 - 2cosx$ and $q = 3sinx + 1$ show that $9p^2 + 4q^2 - 18p - 8q - 23 = 0$

(3) Prove each identity

(a) $\sin(90 - x)\tan(x) \equiv \sin(x)$

(b) $\frac{(\sin(x)+\cos(x))^2}{\sin(x)\cos(x)} \equiv 2 + \frac{1}{\sin(x)\cos(x)}$

(c) $tanA + sinA \equiv \frac{sinA(1+cosA)}{cosA}$

(d) $\frac{cos^4x - sin^4x}{cos^2x - sin^2x} \equiv 1$

(e) $sinx\sqrt{1 + tan^2x} \equiv tanx$

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(54) Solving Basic Trigonometric Equations

WORKING AT D/E

(1) Find the 2 solutions to the equations in the interval $0 \leq x \leq 360$ for each of the following equations:
(a) $\sin(x) = 0.5$
(b) $\cos(x) = 0.5$
(c) $\cos(x) = \frac{\sqrt{3}}{2}$
(d) $\cos(x) = 0$
(e) $\sin(x) = \frac{\sqrt{2}}{2}$

(2) Find the 2 solutions to the equations in the interval $0 \leq x \leq 360$ for each of the following equations. Round answers to 1 decimal place where appropriate.
(a) $\sin(x) = 0.2$
(b) $\cos(x) = \frac{-\sqrt{2}}{2}$
(c) $\cos(x) = 0.65$
(d) $\sin(x) = -0.5$
(e) $\tan(x) = -\sqrt{3}$
(f) $\tan(x) = -2$

(3) Solve the equation $2\sin(x) - 1 = 0$ for $0 < x < 720$

WORKING AT B/C

(1) Solve each equation for $-180 \leq x \leq 180$ giving answers to 1 decimal place where appropriate. For the equations with no solutions, explain why there are no solutions.

(a) $4\sin(x) = 2$
(b) $\cos(x) + 1 = 0.5$
(c) $5\cos(x) = 1$
(d) $3 + \cos(x) = 0$
(e) $2\sin(x) = -\sqrt{3}$
(f) $\tan(x) + 2 = 1$
(g) $3\tan(x) = -\sqrt{3}$

(2) (a) Write down an identity for $\tan(x)$ involving $\sin(x)$ and $\cos(x)$.

(b) Hence, solve the equation $5\sin(x) = 4\cos(x)$ for $0 \leq x \leq 360$ giving your answers to 1 decimal place.

(3) (a) Write down the number of solutions to the equation $x^2 = 3$

(b) Using your answer to part (a) or otherwise, show that there are 4 solutions to the equation $tan^2 x = 3$ for $0 \leq x \leq 360$ giving the value of each.

WORKING AT A*/A

(1) (a) The equation $\sin(x) = a$ has 3 solutions in the interval $-180 \leq x \leq 180$. Write down the value of a
(b) The equation $\sin(x) = b$ has no solutions in the interval $-180 \leq x \leq 180$. Find the value sets of values of b.
(c) The equation $\cos(x) = c$ has 2 solutions in the interval $90 \leq x \leq 270$. Find the set of values of c.

(2) Solve the equation $\frac{cosx}{sinx} = 0.1$ for $-180 \leq x \leq 180$ giving your answers to 1 decimal place.

(3) (a) Write down the number of solutions to the equation $ksin(x) = k$ where k is a positive constant for $90 < x \leq 360$
(b) The equation $\cos(x) = p$ where p is a constant has no solutions for $-90 \leq x \leq 90$. Find the set of values of p

(c) Find the maximum number of solutions to the equation $cos^2 x = n$ where n is a positive content for $0 < x \leq 360$

(d) How many solutions are there to the equation $\tan(x) = r$ where r is a negative constant in the interval $0 \leq x \leq 360$?

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(55) More Challenging Trigonometric Equations

WORKING AT D/E

(1) Find the **4 solutions** to the equation

$$\sin(2x) = \frac{\sqrt{3}}{2}$$

in the interval $0 \leq x \leq 360$

(2) Find the solutions to each equation in the interval $0 \leq x \leq 360$. Give each answer to 1 decimal place where appropriate.
(a) $\cos(x + 30) = 0.5$
(b) $\tan(3x) = 1$
(c) $\sin(x - 60) = 0.1$
(d) $\tan(x + 45) = 0.85$
(e) $\cos(4x) = 0.4$
(f) $\sin(0.5x) = 1$
(g) $4\cos(x - 10) = 0.4$

(3) Show that the solutions to the equation $\cos(2x - 60) = 0.5$ in the interval $0 \leq x \leq 360$ are $x = 60, 180, 240$ and 360^o

WORKING AT B/C

(1) Find the solutions to each equation in the interval $0 \leq x \leq 360$. Give each answer to 1 decimal place where appropriate.
(a) $\cos(2x + 30) = \frac{\sqrt{3}}{2}$
(b) $\sqrt{3}\tan(x - 25) = 1$
(c) $\sin(3x - 30) = -0.5$
(d) $\cos(3x) = -1$
(e) $\cos(x - 16) = -0.25$
(f) $\sin(4x - 60) = -0.85$
(g) $5\cos(0.5x) = 0.4$

(2) (a) Write tan $(3x)$ in terms of sin and cos.

(b) Hence solve the equation $\sin(3x) = \cos(3x)$, $-180 \leq x \leq 180$.

(3) Show that there are 4 solutions to the equation $4sin^2x = 1$ in the interval $0 \leq x \leq 360$

WORKING AT A*/A

(1) (a) Solve the equation
$\sqrt{3}\sin(2x + 30) = \cos(2x + 30),\ -180 \leq x \leq 0$

(b) Solve the equation $4sin^2(3\theta - 45) = 1$ in the interval $-180 \leq \theta \leq 180$

(2) The equation $\sin(ax - b) = \frac{\sqrt{3}}{2}$ where a and b are positive constants has the solutions $x = 22.5^o$ and $x = 37.5^0$ for $-90 \leq x \leq 90$.
Find possible values of a and b.

(3) Solve the equation $(\tan 3x)(2\cos x + 5) = 0$, $-180 \leq x \leq 180$.

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(56) Using Identities to Solve Trig Equations

WORKING AT D/E

(1) Show that the only solutions to the equation

$$\sin(x) = \cos(x), \quad 0 < x \le 360$$

are $x = 45^o$ and $x = 225^o$

(2) (a) Using the identity $\sin^2 A + \cos^2 A \equiv 1$, show that $\cos^2 x + \sin x = 1$ can be written as $\sin x(\sin x - 1) = 0$
(b) Hence, find the 4 solutions to the equation $\cos^2 x + \sin x = 1, \quad 0 \le x \le 360$

(3) (a) Show that $2\sin^2 x - 5\sin x - 3 = 0$ can be written as $(A\sin x + B)(\sin x + C) = 0$ where A, B and C are integers to be found.
(b) Hence, find the 2 solutions to the equation $2\sin^2 x - 5\sin x - 3 = 0$, for $0 \le x \le 360$

WORKING AT B/C

(1) (a) Show that equation

$$8\sin^2 x - 10\cos x - 1 = 0$$

can be written as

$$(4\cos x + 7)(2\cos x - 1) = 0$$

(a) Hence, find the solutions to the equation $8\sin^2 x - 10\cos x - 1 = 0, \quad 0 < x < 360$

(2) Solve the equation $3\tan^2 A - 2\tan A = 1$ in the interval $-180 < A < 180$ giving your answers to 1 decimal place where appropriate.

(3) Find the 5 solutions to the equation

$$\tan x = 2\sin x$$

in the interval $0 \le x \le 360$

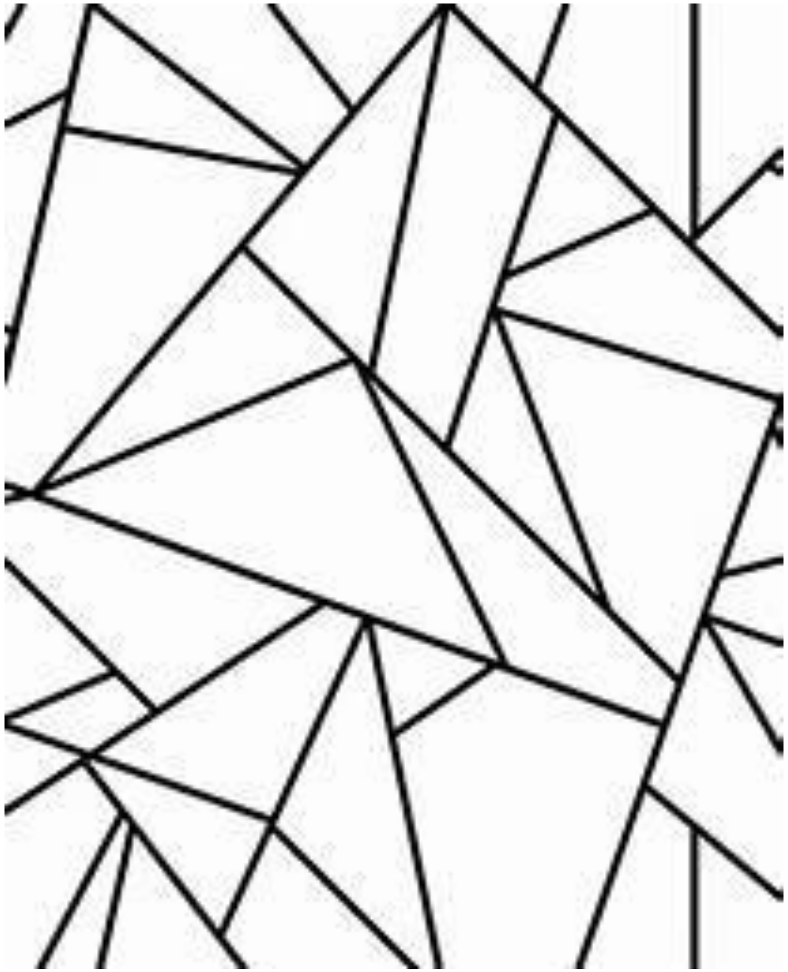
WORKING AT A*/A

(1) (a) Solve the equation $\sin(\theta - 20) = \sin(\theta)$, $0 < \theta < 360$
(b) Solve the equation $\frac{1+\sin^2 4\theta}{\sin 4\theta} = 2$, in the interval $-180 < \theta < 180$

(2) Show that there are no solutions to the equation $8\sin^2 x - 22\cos x - 23 = 0$ when $x \in R$

(3) (a) Prove $\cos^2 A + (1 + \sin A)^2 \equiv 2(1 + \sin A)$
(b) Hence, solve the equation $\cos^2 2x + (1 + \sin 2x)^2 = 0, \quad -180 < x < 180$

Vectors



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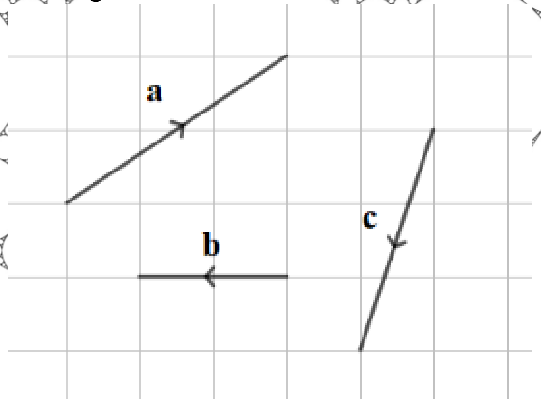
(57) Vectors (Introduction)

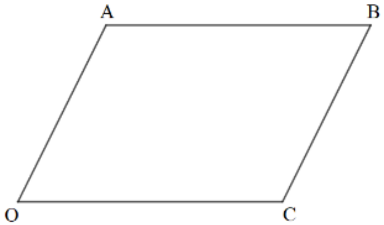
WORKING AT D/E

(1) The diagram shows the vectors **a**, **b** and **c** below.



On squared paper draw the vectors:
(i) **a** + **b** (ii) **c** – **b** (iii) 2**b** - **a**

(2) The diagram below shows the parallelogram $OABC$. $\overrightarrow{OA} = a$ and $\overrightarrow{OC} = c$.
X is the midpoint of OA and Y is the midpoint of OC



(a) Find an expression in terms or a and c for:
(i) $\overrightarrow{OB}$ (ii) $\overrightarrow{AX}$ (iii) $\overrightarrow{AY}$
(b) Show that the lines BC and OA are parallel.

WORKING AT B/C

(1) Given that the vectors 9**a** + p**b** and 2**a** + 6**b** are parallel, find the value of p.

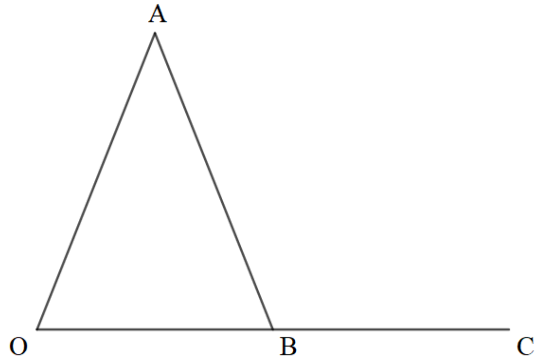
(2) Which of the following vectors are parallel to the vector **a** + **b** ?
(i) 9(**a** + **b**)
(ii) -3**a** + 3**b**
(iii) **b – a**
(iv) 0.5**a** + 0.5**b**
(v) -(**a** + **b**)

(3) $OABC$ is a rectangle. $\overrightarrow{OA} = p$ and $\overrightarrow{OC} = 2q$
The point X lies on OC such that $OX: XC = 1: 3$
The point Y lies on CB such that $CY: YB = 3: 1$

Prove, using vectors, that the line OB and the line XY are parallel.

WORKING AT A*/A

(1) The diagram below shows triangle OAB.
OBC is a straight line, $OA = AB$ and $OB = BC$
$\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$
The point X lies on OA is such that $OX: XA = 2: 1$
The point Y lies on AB such that $BY: YA = 1: 2$



(a) Show that the line XYC is not a straight line. You must show full workings.

(b) Find a vector $\overrightarrow{OD}$ such that XYD is a straight line. You must show full workings.

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(58) Vector Notation (Column and i and j form)

WORKING AT D/E

(1) Given that $\mathbf{a} = \mathbf{i} - 2\mathbf{j}$ and $\mathbf{b} = 4\mathbf{i} + 5\mathbf{j}$, find, in terms of **a** and **b:**

(i) $\mathbf{a} + \mathbf{b}$ (ii) $\mathbf{a} - \mathbf{b}$ (iii) $2\mathbf{a} + 3\mathbf{b}$

(iv) $4\mathbf{a}$ (v) $-2\mathbf{b}$ (vi) $3(\mathbf{a} - \mathbf{b})$

(2) Given that $\mathbf{a} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ find column vectors for:

(i) $\mathbf{a} + 2\mathbf{b}$ (ii) $3\mathbf{a} - \mathbf{b}$ (iii) $2\mathbf{a}$

(iv) $4\mathbf{b}$ (v) $-3\mathbf{c}$ (vi) $5(\mathbf{a} - \mathbf{b})$

(3) In the triangle OAB, $\overrightarrow{OA} = 2\mathbf{p} - 3\mathbf{q}$ and $\overrightarrow{OB} = \mathbf{p} + 7\mathbf{q}$

Find an expression in terms of **a** and **b** for $\overrightarrow{AB}$.

WORKING AT B/C

(1) $\mathbf{a} = \begin{pmatrix} p \\ 6 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 4 \\ q \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

Given that $3\mathbf{a} + 2\mathbf{b} = 5\mathbf{c}$, find the values of p and q.

(2) In the triangle OAB, $\overrightarrow{OA} = 9\mathbf{p} + 2\mathbf{q}$ and $\overrightarrow{AB} = 5\mathbf{p} - 3\mathbf{q}$

Find an expression in terms of **a** and **b** for $\overrightarrow{OB}$.

(3) Given that $\mathbf{a} = \begin{pmatrix} p \\ -4 \end{pmatrix}$ is parallel to $\mathbf{b} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$ find the value of p.

WORKING AT A*/A

(1) Given that the resultant of the vectors $\mathbf{a} = \mathbf{i} - 2\mathbf{j}$ and $\mathbf{b} = p\mathbf{i} + 2p\mathbf{j}$ is parallel to the vector $\mathbf{c} = 4\mathbf{i} + 5\mathbf{j}$.

(a) Find, the value of p as a simplified fraction.

(b) Which has the greatest magnitude, the resultant of **a** and **b** or the vector **c**? You must show workings.

(2) $\mathbf{a} = 4\mathbf{i} + 6\mathbf{j}$ and $\mathbf{b} = -4\mathbf{i} + 10\mathbf{j}$

Given that $\mathbf{a} + \mu\mathbf{b}$ is parallel to the vector $\mathbf{i} + \mathbf{j}$, find the exact value of μ.

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(59) Vectors (Magnitude and Direction)

WORKING AT D/E

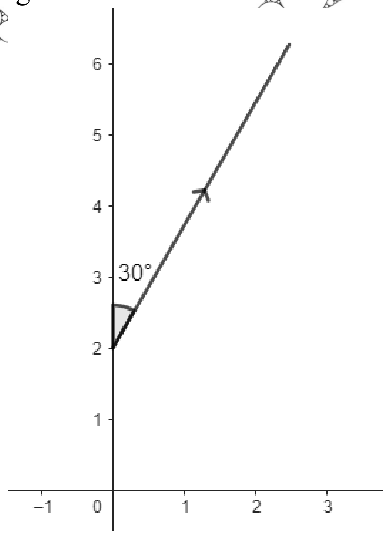
(1) Given that $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ **and** $\mathbf{b} = -4\mathbf{j}$

(a) Show that $|\mathbf{a}| = \sqrt{13}$

(b) Find the resultant of **a** and **b** in the form a**i** + b**j**

(c) Find the modulus of the resultant in the form $\sqrt{p}$

(2) The diagram below shows a vector with length 5



The vector makes an angle of 30^o with the vector **j.**

Show that the vector can be written as $\frac{\mathbf{5}}{\mathbf{2}}\mathbf{i} + \frac{\mathbf{5\sqrt{3}}}{\mathbf{2}}\mathbf{j}$

(3) Find the angle the vector $2\mathbf{i} + 7\mathbf{j}$ makes with the vector **i.**

WORKING AT B/C

(1) A vector has magnitude 8 units and makes an angle of 30^o with the vector **i**. Find the vector in the form a**i** + b**j**, giving **a** as an exact value.

(2) $\mathbf{a} = -3\mathbf{i} + 4\mathbf{j}$

(a) Find a unit vector in the direction of **a.**

(b) Find the angle the vector makes with the vector **j**

(3) Given that $|\mathbf{i} + p\mathbf{j}| = 5\sqrt{2}$, find the possible values of p.

WORKING AT A*/A

(1) Vector **a** has magnitude 4 and makes an angle of $\emptyset$ with the vector **i** .

Given that $\sin\emptyset = \frac{12}{13}$, find the horizontal component of the vector in the form $b\mathbf{i}$.

(2) In triangle OAB, $\overrightarrow{OA} = 2\mathbf{i} + 8\mathbf{j}$ and $\overrightarrow{OB} = 6\mathbf{i} + 3\mathbf{j}$

(a) Find the vector $\overrightarrow{AB}$ in the form p**i** +q**j**

(b) Show that the perimeter of triangle OAB is 21.4 units to one decimal place.

(c) Find the area of the triangle to 3 significant figures.

(3) Given that the vector $\mathbf{a} = 3\mathbf{i} + p\mathbf{j}$ makes an angle of 30^o with the vector **j**, find the value of the constant p.

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(60) Vectors (Position and Direction Vectors)

WORKING AT D/E

(1) Points A and B have coordinates $(3,7)$ and $(4 -8)$ respectively.

(a) Write down the position vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

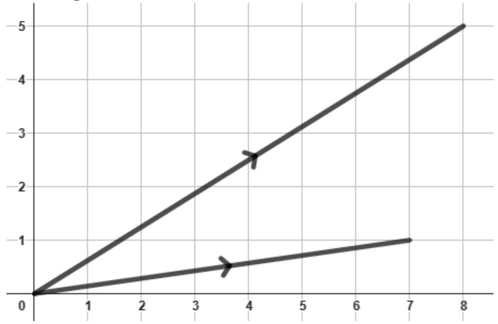
(b) Find the direction vector $\overrightarrow{AB}$.

(c) Hence, write down the vector $\overrightarrow{BA}$.

(d) Find the modulus of $\overrightarrow{AB}$ in exact form.

(e) Find the angle the vector $\overrightarrow{OA}$ makes with the positive x axes.

(2) The diagram shows the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$.



(a) Given that $|\overrightarrow{OA}| = 5\sqrt{2}$, find the coordinates of the vector B.

(b) Explain why $\overrightarrow{BA}$ can be written as $\begin{pmatrix}1\\4\end{pmatrix}$.

(c) Find $|\overrightarrow{AB}| =$

(3) Given that $\overrightarrow{OC} = \begin{pmatrix}-4\\8\end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix}2\\6\end{pmatrix}$, find $\overrightarrow{OD}$ in the form $\begin{pmatrix}p\\q\end{pmatrix}$

WORKING AT B/C

(1) Given that $\overrightarrow{OA} = \begin{pmatrix}-4\\8\end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix}-3\\-2\end{pmatrix}$,

(a) Find $|\overrightarrow{OB}|$ in the form $\sqrt{p}$

(b) Find the angle $\overrightarrow{OB}$ makes with the vector **-i** .

(c) $OBAC$ is a parallelogram. Find the coordinates of C.

(2) $OABC$ is a kite.

$|\overrightarrow{OA}| = |\overrightarrow{OC}|$ and $|\overrightarrow{AB}| = |\overrightarrow{BC}|$

$\overrightarrow{OA} = -2\mathbf{i} - 6\mathbf{j}$ and $\overrightarrow{AB} = 2\mathbf{i} - 4\mathbf{j}$

(a) Find $\overrightarrow{OC}$

(b) Find the area of the kite $OABC$

(3) $\overrightarrow{OC} = \begin{pmatrix}2\\1\end{pmatrix}$ and $\overrightarrow{OD} = \begin{pmatrix}p\\q\end{pmatrix}$

Given that $\overrightarrow{DC} = \begin{pmatrix}8\\-3\end{pmatrix}$, find $|\overrightarrow{OD}|$

WORKING AT A*/A

(1) A circle has equation $x^2 + y^2 = 25$. The point P lines on the circle and has position vector $\overrightarrow{OP} = \begin{pmatrix}6m\\8m\end{pmatrix}$ where m is a constant. Find the possible coordinates of the point P.

(2) $\overrightarrow{OA} = -10\mathbf{i}$ and $\overrightarrow{OB} = -6\mathbf{i} - 10\mathbf{j}$

(a) Find $|\overrightarrow{OA}|$ and $|\overrightarrow{OB}|$

(b) Prove that ΔOAB is not an isosceles triangle.

(c) Find the area of ΔOAB

(3) $\overrightarrow{OA} = 5\mathbf{i} - 6\mathbf{j}$ and $\overrightarrow{AB}$ is parallel to the vector $\mathbf{j}$. Given that $\overrightarrow{OB} = p\mathbf{i}$ where p is a constant, find $\overrightarrow{AB}$.

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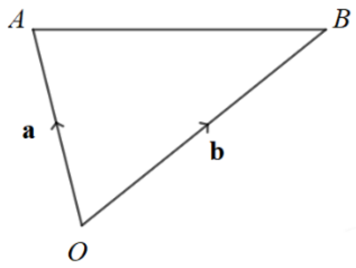
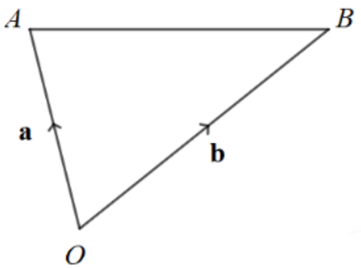
(61) Vector Geometry

WORKING AT D/E

(1) The diagram below shows the triangle OAB.



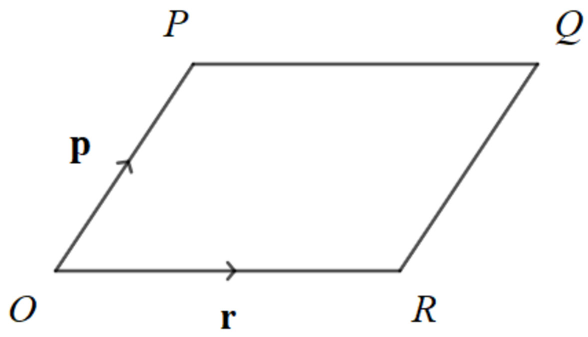
(a) Write down $\overrightarrow{OA}$ and $\overrightarrow{OB}$ in terms of **a** and **b**.
(b) Hence, find $\overrightarrow{AB}$ in terms of **a** and **b**.
The point X lies on AB such that $AX:XB$ is 1:2
(c) Mark the point X on the diagram below.



(d) Show that $\overrightarrow{OX} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$
The point Y lies on OB such that $OY:YB$ is 1:2
(e) Prove, using vectors, that $\overrightarrow{OA}$ and $\overrightarrow{YX}$ are parallel.
The point Z lies on OA such that $\overrightarrow{YZ}$ and $\overrightarrow{AB}$ are parallel.
(f) Find $\overrightarrow{OZ}$.

WORKING AT B/C

(1) The diagram below shows the parallelogram $OPQR$.

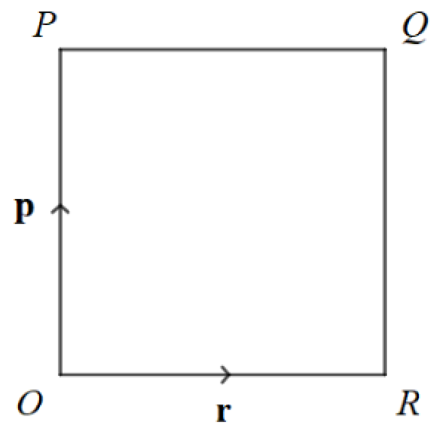


(a) Write down $\overrightarrow{OQ}$ in terms of **p** and **r**.
Point X is the midpoint of the line OP of and point Y is the midpoint of the line RQ.
(b) Prove, using vectors that $\overrightarrow{PQ}$ and $\overrightarrow{YX}$ are the same.
(c) Z is the midpoint of the line OQ. Use vectors to show that Z is also the midpoint of the line PR.

Given further, that $\mathbf{p} = 2\mathbf{i} + 6\mathbf{j}$, $\mathbf{r} = q\mathbf{i}$, where q is a constant, and that the area of the parallelogram is 45 units,
(d) Find the exact value of q.

WORKING AT A*/A

(1) The diagram below shows the square $OPQR$.



The point X lies on OR such that $OX:XR$ is 1:3
The point Y lies on RQ such that $RY:YQ$ is 3:1
The point Z is the midpoint of the line OP
(a) Using vectors, find the ratio $ZQ:XY$

Given further that $\overrightarrow{OZ} = 4\mathbf{j}$,
(b) Find $|\overrightarrow{OR}|$ as a simplified surd.
(c) Write down the angle $\overrightarrow{OR}$ makes with $\overrightarrow{OZ}$

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(62) Application of Vectors

WORKING AT D/E

(1) Alan travels 6m due west from a fixed point O to the point A. Alan then moves directly south from A 8m to the point B.
(a) Find the position vectors $\overrightarrow{OA}$ and $\overrightarrow{AB}$ using **i** and **j** notation.
(b) Hence, find $\overrightarrow{OB}$.
(c) Show that $|\overrightarrow{OB}| = 10$m.
(d) Find the bearing of B from O.

The point C is 22m due east of B.
(e) Find $\overrightarrow{OC}$ using **i** and **j** notation.

Alan walks the perimeter of the triangle OBC.
(f) Find the distance he walks in total in exact form.

Beryl is standing 14m due north of the point C.
(g) Find the bearing of C from O.

Beryl now walks back to O from C at a constant speed of $2.4ms^{-1}$.
(h) Show that it will take approximately 7 seconds for Beryl to reach O from C.

WORKING AT B/C

(1) Alan walks from the fixed point O to the point A where $\overrightarrow{OA} = \begin{pmatrix} 5\sqrt{3} \\ 5 \end{pmatrix}$ m

(a) Show that the bearing of A from O is 060^0.

(b) Find the distance Alan walks.

Alan now walks directly south to the point B.

(c) Given that B is directly east of O, write down $\overrightarrow{OB}$ in the form a**i** .

From B, Alan walks to the point C.

(d) Given that $\overrightarrow{BC} = \begin{pmatrix} -12\sqrt{3} \\ 10 \end{pmatrix}$ m, find the bearing of C from O.

(e) Alan now walks back to O. Find the distance OC to 3 significant figures.

WORKING AT A*/A

(1) Beryl walks 20m on a bearing of 045^0 from a fixed point O to the point A.
(a) Find $\overrightarrow{OA}$ in the form $\begin{pmatrix} p \\ q \end{pmatrix}$ where p and q are exact values.

Beryl now walks from the point A to the point B.
(b) Given that B is 10m from A and on a bearing of 135^o from A, find $\overrightarrow{OB}$ in column form.

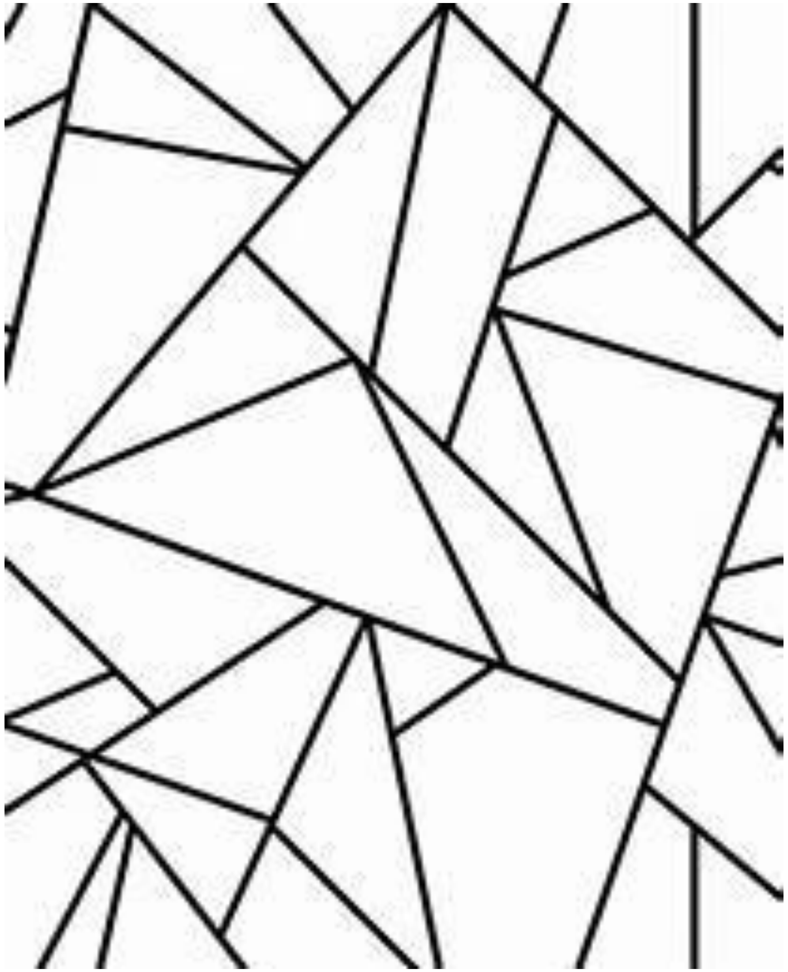
(c) Find $|\overrightarrow{OB}|$.

(d) Find the bearing of B from O.

Point D is 1m from O.
(e) Given that $\overrightarrow{OD}$ is in the direction to the vector -3**i** – 4**j**, find $\overrightarrow{OD}$ in the form (**a**i + **b**j)m where **a** and **b** are simplified fractions.

The point E is due east of D and south of O.
(f) Write down $\overrightarrow{OE}$ using **i** and **j** notation.

Differentiation



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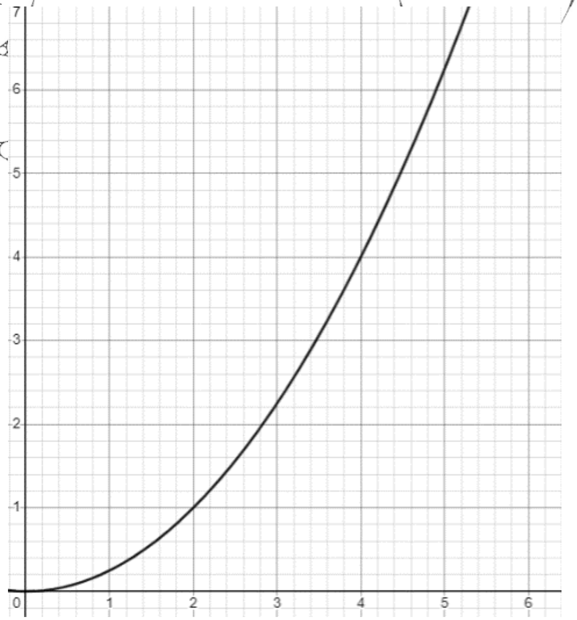
(63) Differentiation (Gradients of Curves)

WORKING AT D/E

(1) The diagram below shows part of the curve of an equation.



(a) Draw a chord from the point with x coordinate 2 to the point with x coordinate 4.

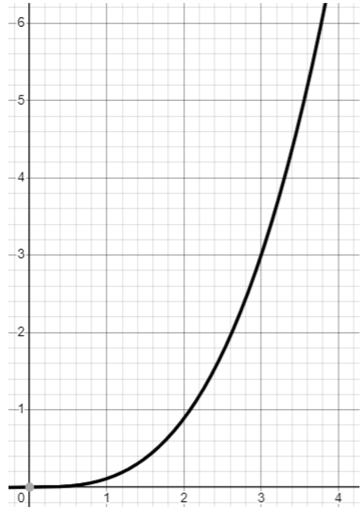
(b) Hence show that the gradient of the chord is $\frac{3}{2}$

(c) Draw a tangent to the curve at the point with x coordinate 4.

(d) Find an estimate for the gradient of the tangent at the point with x coordinate 4.

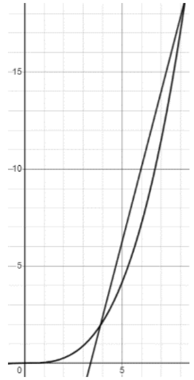
WORKING AT B/C

(1) The diagram below shows part of the curve of an equation.



Estimate the gradient of the tangent to the curve at the point with coordinates (3,3)

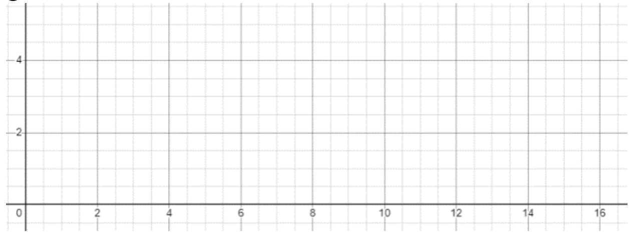
(2) Alan draws a straight line to estimate the gradient of the curve below at the point (4,2).



Explain what he could do to get a better estimate to the gradient.

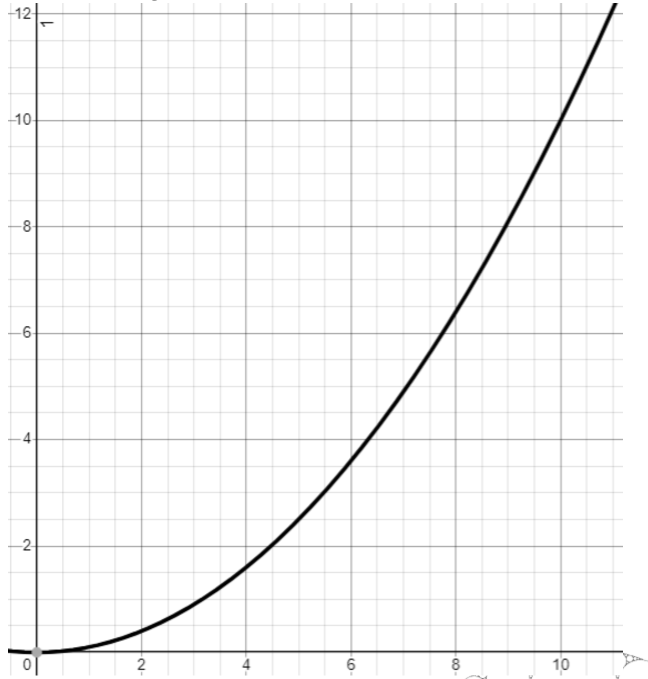
WORKING AT A*/A

(1)(a) Plot the graph of $y = \sqrt{x}$, $0 \leq x \leq 16$ on a grid like the one shown below.



(b) Explain what happens to the gradient of the curve as $x \to \infty$.

(2) On the diagram below, find a point where the curve has a gradient of ~ 1



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(64) Differentiation from 1st Principles

WORKING AT D/E

(1) Prove, from first principles, the derivative of x^2 is $2x$.

WORKING AT B/C

(1) Prove, Prove, from first principles, the derivative of $4x^2 - 3x$ is $8x - 3$.

WORKING AT A*/A

(1) Using first principles, find the derivative of x^4.

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(65) Differentiating x^n (Basic Powers of x)

WORKING AT D/E

(1) Find an expression for $\frac{dy}{dx}$ for each of the following:

(a) $y = x^4$ (b) $y = x^7$ (c) $y = 2x^3$

(d) $y = 5x^4$ (e) $y = x^{\frac{3}{2}}$ (f) $y = x^{-1}$

(g) $y = -4x^7$ (h) $y = 8x^{\frac{1}{4}}$ (i) $y = \sqrt{x}$

(2) Find an expression for $f'(x)$ for each of the following:

(a) $f(x) = x^{\frac{4}{5}}$ (b) $f(x) = 3x^{\frac{1}{3}}$ (c) $f(x) = \frac{6}{x}$

(d) $f(x) = -x^{-\frac{2}{5}}$ (e) $f(x) = \frac{1}{2x^2}$

(3) Find an expression for $\frac{dx}{dt}$ given $x = 8\sqrt[4]{t}$

WORKING AT B/C

(1) Find a simplified expression for $\frac{dy}{dx}$ for each of the following:

(a) $y = x\sqrt{x}$ (b) $y = \frac{x^7}{2x}$ (c) $y = \frac{4}{3\sqrt[5]{x}}$

(2) Find a simplified expression for $f'(x)$ for each of the following:

(a) $f(x) = \left(2x^{\frac{7}{2}}\right)^4$ (b) $f(x) = \frac{8x}{\sqrt[4]{x^3}}$

(3) Find an expression for $\frac{dP}{dt}$ given $P = 0.5t\sqrt{t}$

WORKING AT A*/A

(1) Find a simplified expression for $h'(t)$ given that $h(t) = \sqrt[4]{16t^8} \times \frac{3}{t^{0.25}}$

(2) Find the gradient of the curve with equation $y = 2\sqrt[4]{t}$ when $t = 16$.

(3) $f(x) = 2x^2$
Find the value of x for which $f'(x) = 64$

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(66) Differentiation (Quadratic Expression)

WORKING AT D/E

(1) Find a simplified expression for $\frac{dy}{dx}$ for each quadratic equation:

(a) $y = x^2 + 3x$ (b) $y = x^2 - 2x + 4$

(c) $y = -x^2 + 6x - 3$ (d) $y = 4x^2 - 3x$

(2) Find an expression for $f'(x)$ for each of the following quadratic equations:

(a) $f(x) = 5x^2 - x$ (b) $f(x) = -3x^2 + 2$

(3) Given that $f(x) = 4x^2 + 2x - 7$

(a) Find the gradient of the curve $y = f(x)$ when $x = 2$

(b) Find the value of x when $f'(x) = 34$

WORKING AT B/C

(1) Find a simplified expression for $\frac{dy}{dx}$ for each equation:

(a) $y = x(x-4)$ (b) $y = (x-3)(x+4)$

(c) $y = (2x-1)(3x+5)$ (d) $y = (x-3)^2$

(2) Find an expression for $g'(x)$ for each of the following equations:

(a) $g(x) = 6x(x-4)$ (b) $g(x) = (4x-3)^2$

(3) Given that $y = (4-5x)^2$ find the value x for which $\frac{dy}{dx} = 5$

WORKING AT A*/A

(1) $f(x) = x^2 + px + q$

Given that $f(2) = 18$ and $f(-3) = -27$

(a) Find the value of the constants p and q.

The curve with equation $y = f(x)$ has gradient -8 at the point (a, b)

(b) Find the value of a and the value of b.

(c) Find the coordinates of the point where the tangent to the curve is parallel to the line $y = 0$

(2) The graph of $x = 4t^2 - 8t$ at the point (p, q) has gradient -40.

Find the value of p and the value of q.

(3) Find the coordinates of the point on the graph of $y = -(2-3x)^2$ where the gradient is 6

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(67) Differentiation (Multiple Terms)

WORKING AT D/E

(1) Find an expression for $\frac{dy}{dx}$ for each of the following:

(a) $y = x^7 - 3x$ (b) $y = x^7(x-1)$

(c) $y = 4x + \sqrt{x} + 1$ (d) $y = x^{\frac{3}{2}}(x-3)$

(2) Find an expression for $f'(x)$ for each of the following:

(a) $f(x) = 7x^{\frac{2}{5}} - \frac{4}{x}$ (b) $f(x) = x^{\frac{6}{11}}(2x-3)$

(c) $f(x) = \frac{4}{x}(6x+2)$ (d) $f(x) = -3x^{-\frac{1}{5}} + 8x^{\frac{1}{3}}$

(3) Given that $x = t\sqrt{t} + \frac{10}{t^2}$, show that $\frac{dx}{dt} = \frac{3}{2}\sqrt{t} - \frac{20}{t^3}$

WORKING AT B/C

(1) A curve has a stationary point when $\frac{dy}{dx} = 0$

Find the x coordinate of the two stationary points on the curve with equation $y = 8x + \frac{1}{x}$

(2) Give that $f(x) = 8x^{\frac{3}{4}} - 2x^{0.5}$

Show that $f'(16) = \frac{11}{4}$

(3) Given that $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x$

(a) Show that $\frac{dy}{dx} = (x+3)(x-2)$

(b) Hence, find the 2 values of x for which the curve $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x$ has a stationary point.

WORKING AT A*/A

(1) $y = -\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}x^2 - 42x, \quad x \in R, x > 0$

Find the coordinates of the only point on the curve where $\frac{dy}{dx} = 0$, giving the y coordinate as an exact fraction.

(2) $f(x) = 12 - x^{0.5}$

Use differentiation to show that the curve with equation $y = f(x)$ doesn't have a stationary point.

(3) The curve with equation $y = ax^2 + bx + c$ has:

- A stationary point when $x = \frac{-3}{8}$
- Crosses the y axis when $y = 1$
- Has gradient -5 when $x = -1$

(a) Find the values of a, b and c.

(b) Sketch the curve of $y = ax^2 + bx + c$

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(68) Differentiation (Gradients, Tangents and Normals)

WORKING AT D/E

(1) A curve has equation $y = 4x^3 + 2x + 1$
(a) Find the value of y when $x = 1$
(b) Find an expression for $\frac{dy}{dx}$
(c) Find the gradient of the curve at the point where $x = 1$
(d) Hence, show that the equation of the tangent to the curve at the point $(1,7)$ is $y = 14x - 7$
(e) Write down the gradient of the normal at the point $(1,7)$.
(f) Hence, show that an equation of the normal at $(1,7)$ is $x + 14y = 99$

(2) $y = 4x^3 - 5x^2 + 2$
(a) Find the equation of the tangent to the curve at the point with x coordinate 2. Give your answer in the form $y = mx + c$
(b) Find an equation of the normal to the curve at the point with x coordinate 3.

(3) $y = x^2 + 6x$
Find the equation of the tangent to the curve when the gradient is 3 in the form $y = mx + c$.

WORKING AT B/C

(1) (a) Find the equation of the tangent to the curve with equation $y = \frac{1}{x}$ at the point where $x = 2$ giving your answer in the form $ax + by = c$.

(b) Show that the normal to the curve at the point $(4, \frac{1}{4})$ can be written as $y = 16x + c$ where c is an exact fraction to be found.

(2) The curve with equation $y = 2x^5 + x$ has a tangent at the point (p, q) where p and q are positive constants.

Given that the tangent is parallel to the line with equation $y = 11x - 3$, find the values of p and q.

(3) The normal to the curve with equation $y = x^2$ at the point with x coordinate -3 crosses the x axis at A and y axis at B.

Show that $AB = \frac{19\sqrt{37}}{2}$

WORKING AT A*/A

(1) The normal to the curve with equation $y = 2x\sqrt{x}$ is parallel to the line with equation $36y + 2x - 3 = 0$.
Find where the normal crosses the x axis.

(2) The normal to the curve with equation $y = -x(x-3)$ at the point $P\ (2, y)$ intersects the curve at the point P and the point Q.
Find the coordinates of the point Q.

(3) (a) Find the coordinates of the point P on the curve with equation $y = 2x^{0.5} + 2x - 8,\ \ x > 0$ where the tangent at P is parallel to the line with equation $12x - 2y = 7$
(b) The tangent to the curve at P crosses the x axis at A and y axis at B. Find the area of ΔAOB where O is the origin.

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(69) Differentiation (Increasing and Decreasing Functions)

WORKING AT D/E

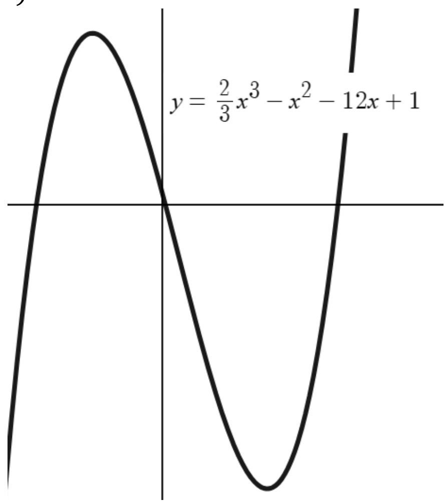
(1) Showing that the interval for which the function $f(x) = 3x^2 - 12x + 1$ is increasing is $x > 2$.

(2) (a) Show that the set of values for which the function $f(x) = \frac{8}{3}x^3 - x^2 - 3x + 9$ is decreasing a decreasing function satisfies the inequality $0 > (4x - 3)(2x + 1)$
(b) Hence, find the set of values for which the function is decreasing.

WORKING AT B/C

(1) $f(x) = ax^3 - x + b,\ a > 0$
(a) Given that $f(x)$ is increasing when $x > 2$, find the value of a.
(b) Explain why the value of b doesn't change the answer to part (a)

(2) $f(x) = \frac{2}{3}x^3 - x^2 - 12x + 1$
The diagram shows part of the curve with equation $y = f(x)$



(a) Show that $f'(x)$ can be written as $f'(x) = 2(x + 2)(x - 3)$
(b) Using your answer to part (a), find the values or set of values for which $f(x)$ is:
(i) Stationary, (ii) A decreasing function
(iii) An increasing function

(3) $f(x) = x + \frac{1}{x}, \quad x \neq 0$
Show that the set of values for which $f(x)$ is increasing is $-1 < x$, $x > 1$.

WORKING AT A*/A

(1) $f(x) = 2x^3 + 5x^2 + 8x + 3$
(a) Show that $f(-0.5) = 0$
(b) Hence factorise $f(x)$
(c) Find $f'(x)$
(d) Show that $f(x)$ is an increasing function for all values of x
(e) Hence sketch the graph of $y = f(x)$ showing where the curve crosses the coordinate axes.

(2) $f(x) = (x + a)(x + b)(x + c)(x + d)$ where a, b, c and d are all different integers.
(a) Write down the number of intervals for which the function is increasing.
(a) Write down the number of intervals for which the function is decreasing.

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(70) Differentiation (Stationary Points)

WORKING AT D/E

(1) $f(x) = 2x^3 + 4x^2$
(a) Find $f'(x)$
(b) Hence, show that the x coordinates of the two stationary points are $x = 0$ and $x = -\frac{4}{3}$
(c) Hence, find the coordinates of the two stationary points.
(d) Find an expression for $f''(x)$
(e) Find $f''(0)$ and $f''(-\frac{4}{3})$
(f) Hence determine the nature of each stationary point.

(2) $y = 4x^5$
(a) Find an expression for $\frac{dy}{dx}$
(b) Hence find the one stationary point on the curve.
(c) By considering the value of $\frac{dy}{dx}$ when $x = -0.01$ and when $x = 0.01$, explain why the stationary point is a point of inflexion.

(3) $y = \frac{4}{3}x^{\frac{3}{2}} - 18x$
(a) Use differentiation to show that the stationary point on the curve has coordinates $(81, -486)$.
(b) Determine the nature of this stationary point.

WORKING AT B/C

(1) $f(x) = (x + 1)(x - 3)(x + 2)$
(a) Find an expression for $f(x)$ in the form $f(x) = Ax^3 + Bx^2 + Cx + D$
(b) Use differentiation to show that the x coordinates of the two stationary points on the curve with equation $y = f(x)$ are $x = \pm\frac{\sqrt{21}}{3}$
(c) Find the y coordinate of each stationary point giving each answer to 3SF.
(d) Determine the nature of each stationary point.
(e) Hence, sketch the curve of $y = f(x)$ labelling each stationary point and the points where the curve crosses the coordinate axes.

(2) A curve has equation $y = (x - 2)(x^2 + 5x + 10)$
(a) Show that the only root of the equation is $x = 2$
(b) Find any stationary points on the curve.
(c) Find an expression for $\frac{d^2y}{dx^2}$
(d) Using your answer to part (c) show that one of the stationary points is a maximum and one is a minimum.
(e) Hence, sketch the curve of $y = (x - 2)(x^2 + 5x + 10)$ labelling each stationary point and the points where the curve crosses the coordinate axes.

WORKING AT A*/A

(1) A curve has equation $y = \frac{x^{\frac{4}{3}} - x}{\sqrt{x}}$, $x > 0$
(a) Find an expression for $\frac{dy}{dx}$ in the form $Ax^n(B + Cx^m)$
(b) Hence, show that the x coordinate of the stationary point is $x = \frac{27}{125}$
(c) Prove that this is a minimum point.

(2) Determine the least value of the function $g(x) = 2x^4 + 64x$

(3) Prove that $f(x) = x^3 - 3x^2 + 18x + 12$ is an increasing function for all values of x.

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(71) Differentiation (Gradient Functions)

WORKING AT D/E

(1) $f(x) = 8x^2 + 4x + 1$
(a) Find an expression for $f'(x)$
(b) Hence sketch the graph of the gradient function showing where the graph crosses the coordinate axes.

(2) $g(x) = 3x^3 + \frac{3}{2}x^2 - 2x + 9$
(a) Find an expression for $g'(x)$
(b) Hence, show that $g(x)$ is stationary when $x = -\frac{2}{3}$ and when $x = \frac{1}{3}$
(c) Sketch the graph of the gradient function of $g(x)$

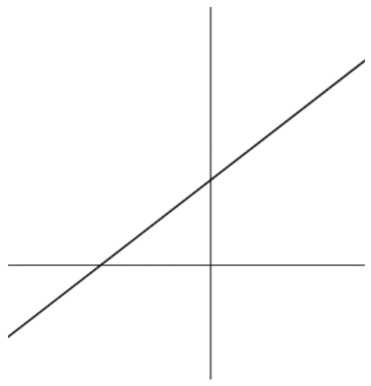
(3) Complete the sentence:
"The graph of the gradient function of a quartic equation will be a__________function"

WORKING AT B/C

(1) $f(x) = 6x^4 + 4x^3 - 12x^2 - 12x + 7$
(a) Find an expression for $f'(x)$
(b) Find the values of x for which $f(x)$ is stationary.
(c) Hence sketch the graph of the gradient function showing where the graph crosses the coordinate axes.

(2) $g(x) = -x^3 + \frac{11}{2}x^2 + 20x - 5$
(a) Show that $g(x)$ is stationary when $x = 5$ and $x = -\frac{4}{3}$
(b) Sketch the graph of the gradient function of $g(x)$ showing where the graph crosses the coordinate axes.

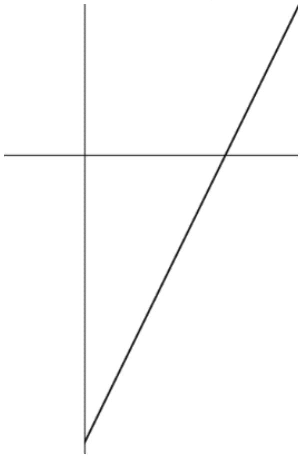
(3) Part of the graph of the gradient function of $h(x)$ is shown below.



Explain why $h(x)$ could be written in the form $h(x) = Ax^2$

WORKING AT A*/A

(1) The graph of the gradient function of $f(x) = Ax^2 + Bx + C,\ x > 0$, is shown below.



(a) Find the set of value of the constant A
(b) Write down where the line crosses the x axis in terms of A and B.
(c) Explain why the set of values of C cannot be determined from the graph.

(2) $g(x) = Ax^3 - Bx$, where A and B are positive constants.

Sketch the graph of the gradient function of $g(x)$ showing where the graph crosses the coordinate axes giving the coordinates in terms of A and B .

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(72) The Applications of Differentiation

WORKING AT D/E

(1) The height of a rocket above the ground (h) in metres after time (t) seconds can be modelled by the equation:

$$h = -t^3 + 2t^2 + 15t, \quad 0 \leq t \leq 3.8$$

(a) Factorise $-t^3 + 2t^2 + 15t$
(b) Hence show that the rocket is only at ground level at the start of the flight.
(c) Find an expression for $h'(t)$
(d) Hence show that the particle is station when $t = 3$
(e) Hence, find the maximum height of the rocket.
(f) Find an expression $h''(t)$
(g) Use your answer to (e) to verify this is a maximum height.
(h) Draw a sketch of $h = -t^3 + 2t^2 + 15t$, $0 \leq t \leq 3.8$

WORKING AT B/C

(1) A piece of wire of length 60cm is bent and made into a rectangle with side lengths x and $2y$.

(a) Show that $2y = 30 - x$

(b) Show that the area (A) of the rectangle can be written as $A = x(30 - x)$

(c) Use differentiation to find the value of x that maximises the area of the rectangle.

(d) Find $\frac{d^2A}{dx^2}$,

(e) Hence, show that this is a maximum value.

(f) Find the maximum area of the rectangle.

(g) Sketch the gradient function $A = x(30 - x)$

Beryl believes there could also be a minimum value for x too.
(h) Explain why she is wrong.

WORKING AT A*/A

(1) The horizontal distance of a car (x) in metres from a fixed point (O) after time (t) seconds can be modelled by the equation

$$x = -t(t - t^{0.5} - 12), \quad 0 \leq t \leq 12$$

(a) State the initial distance of the car from O.

(b) Show that when the car is at its furthest distance from the O, t satisfies the equation:

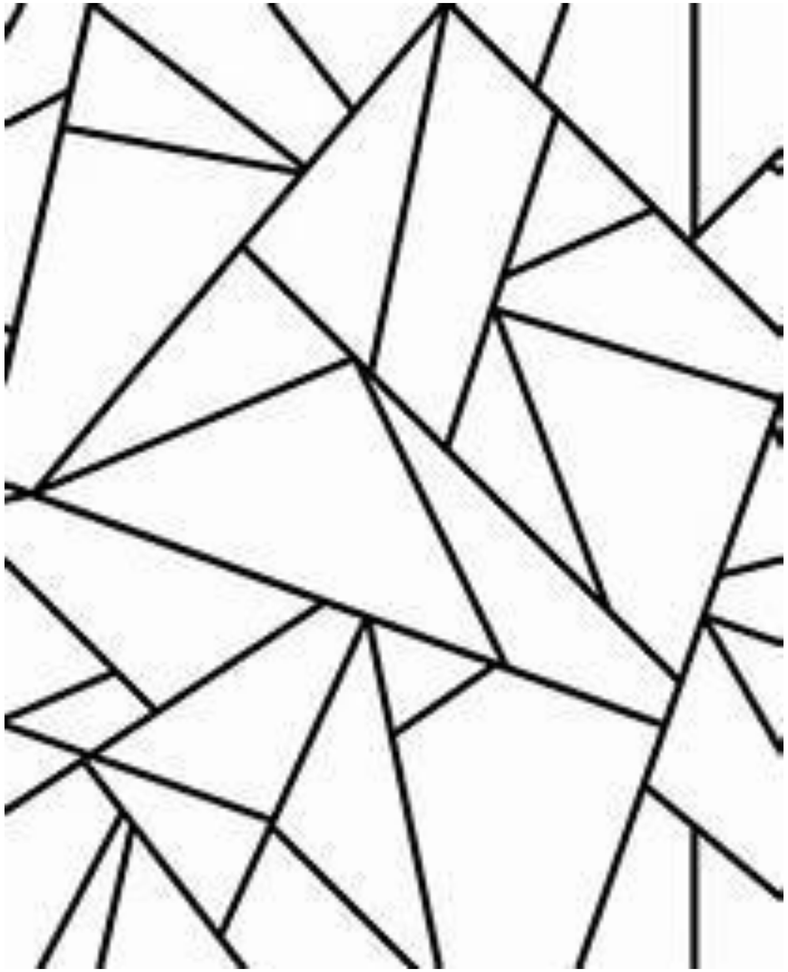
$$0 = A + Bt^{0.5} + Ct$$

Where A, B and C are integers to be found.

(c) Find the maximum distance from O that the car reaches. Give your answer to 3 SF.

(d) Show that the car never returns to O.

Integration



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(73) Integration (Basic Expressions (x^n))

WORKING AT D/E

(1) Find a simplified expression for y, including a constant of integration for each:

(a) $\frac{dy}{dx} = 4x$ (b) $\frac{dy}{dx} = 2x^2$ (c) $\frac{dy}{dx} = 4x^3 - 8x$

(d) $\frac{dy}{dx} = 5x^2 - x + 3$ (e) $\frac{dy}{dx} = \frac{5}{6}x^{\frac{1}{2}}$

(2) Find a simplified expression for $f(x)$, including a constant of integration for each:

(a) $f'(x) = x^{\frac{3}{2}}$ (b) $f'(x) = 5x^{-2}$ (c) $f'(x) = \sqrt{x}$

(3) $\frac{dy}{dx} = (3x+2)^2$

(a) Show that $\frac{dy}{dx}$ can be written in the form $Ax^2 + Bx + C$

(b) Hence find a simplified expression for y.

WORKING AT B/C

(1) Find a simplified expression for y, including a constant of integration for each:

(a) $\frac{dy}{dx} = \frac{2}{x^2} + \sqrt[3]{x}$ (b) $\frac{dy}{dx} = 8x^{-0.25} - x^{2.5}$

(c) $\frac{dy}{dx} = x\sqrt{x}$ (d) $\frac{dy}{dx} = \frac{24}{x^{\frac{2}{3}}} + 3x^{\frac{2}{5}}$

(2) $f'(x) = \frac{x^2 - 3x + 8}{\sqrt{x}}$

(a) Show that $f(x)$ can be written in the form $f'(x) = Ax^p + Bx^q + Cx^r$

(b) Hence, find $f(x)$ giving each coefficient as a simplified fraction.

(3) Given that $\frac{dy}{dx} = \frac{(x^3-1)^2}{x^3}$ show that

$y = \frac{1}{4}x^4 - 2x - \frac{1}{2x^2} + c$

WORKING AT A*/A

(1) $\frac{dy}{dx} = \frac{(\sqrt{x}+2)^2}{x^3}$

Find a simplified expression for y.

(2) $g(x)$ has gradient function $\frac{3}{x\sqrt{x}} - x$.

Find a general solution for $g(x)$.

(3) Given that $f'(x) = (x + x^{\frac{1}{3}})^3$

Find a general solution for $f(x)$ giving each coefficient as a simplified fraction.

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(74) Indefinite Integrals

WORKING AT D/E

(1) Find each, simplifying any coefficients that are fractions:

(a) $\int \sqrt[5]{x^4}\,dx$ (b) $\int (x-1)(x+2)\,dx$

(c) $\int \frac{5}{x^3}\,dx$ (d) $\int 4x^{-7}\,dx$

(2) Given that

$\int (4x^3 + px^2 + q)\,dx = x^4 + 2x^3 + 9x + c$

where p, q and c are constants, find the values of p and q.

(3) Show that $\int \frac{7}{2t^{\frac{1}{3}}}\,dt = \frac{21}{4}t^{\frac{2}{3}} + c$

WORKING AT B/C

(1) Show that $\int y - y^{-2}\,dy = \frac{1}{2}y^2 + \frac{1}{y} + c$

(2) Find $\int \frac{4t^3 - \sqrt{t}}{2t^2}\,dt$ simplifying the coefficients of each term.

(3) Given $\int (Ax+B)^2\,dx = 3x^3 + 6x^2 + 4x + c$ find the positive constants A and B

WORKING AT A*/A

(1) $f(x) = (1-3x)^8$

Given that x is small such that terms in x^3 and higher can be ignored:

(a) Show that an approximation for $f(x)$ can be written in the form $f(x) = P + Qx + Rx^2$

(b) Find an approximation for $\int f(x)\,dx$

(1) Find $\int \left(4 - \frac{\sqrt{t}-1}{t^2}\right) dt$ simplifying the coefficients of each term.

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(75) Integration (Finding *c* and Finding Functions)

WORKING AT D/E

(1) A curve with equation $y = f(x)$ passes through the point (1,2).
Given that $\frac{dy}{dx} = 3x^2 + 4x - 7$, show that
$y = x^3 + 2x^2 - 7x + 6$

(2) (a) Find $\int \left(\frac{4}{3}x^{\frac{1}{2}}\right) dx$

A curve with equation $y = f(x)$ and passes through the point (9,12).
(b) Given that $f'(x) = \frac{4}{3}x^{\frac{1}{2}}$ find $f(x)$.

(3) The gradient function of $g(x) = \frac{2}{x^2}$
Given that the point $(-0.25, 8)$ lies on the graph with equation $y = g(x)$, find an expression for $g(x)$

WORKING AT B/C

(1) A curve has equation $y = f(x)$

Given that $\frac{dy}{dx} = 5x\sqrt{x}$ and that (1,3) is a point the curve, find an expression for $f(x)$.

(2) A curve has equation $y = f(x)$. The point (1,0) lies on the curve.

Given that $f'(x) = 1 - \frac{8}{x^3}$, find $f(x)$ in the form $Ax^n + Bx + C$ where A, B and C are integers and n is a rational fraction.

(3) The gradient function of a curve is given as $\frac{dy}{dx} = 4x^2$

(a) Write down what type of equation the curve has.

(b) Given that the point $(3, 35)$ lies on the curve, draw a sketch of the curve showing where the curve crosses the y axis.

WORKING AT A*/A

(1) Beryl has created a logo for her art project using a computer animation package.
The area (A) of the onscreen logo she designs is such that the rate of change of the area with respect to time (t) is given as $-3t^2 + 6t + 4$

The animation appears on the screen from a dot and disappears 4 seconds later.

(a) Find an equation for the model in the form $A = f(t)$
(b) Find the area of the logo after one second.
(c) Find when the logo is at its largest. Give your answer to 3 S.F.

(2) $x = f(t),\ 0 < t < 5$
(a) Given that $f'(t) = \frac{8t-1}{t^3}$ and when $t = 1, x = 4$, find x when $t = 2$
(b) Write down the set of values for which $f(t)$ is decreasing.
(c) Find the greatest value of $f(t)$,

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(76) Integration (Definite Integrals)

WORKING AT D/E

(1) Without using a calculator, show that

$$\int_1^3 (2x+4)dx = 16$$

(2) Evaluate each of the following. Give your answers as exact fractions where appropriate. You must show full workings:

(a) $\int_0^3 (x^4 + x)\, dx$ (b) $\int_1^4 \left(\frac{5}{x^2}\right) dx$

(c) $\int_4^9 (x^{-0.5} - 1)\, dx$ (d) $\int_1^{25} \left(x^{1/2}\right) dx$

(3) Evaluate $\int_1^8 \left(4 - 3t + \sqrt[3]{t}\right) dt$

WORKING AT B/C

(1) Without using a calculator, show that

$$\int_2^8 \left(2x + \frac{1}{\sqrt{x}}\right) dx = 60 + 2\sqrt{2}$$

(2) Showing full workings, evaluate

$$\int_1^3 \left(\frac{6x^5 + x^3 - 2x}{x}\right) dx$$

WORKING AT A*/A

(1) Given that:

$$\int_n^{4n} (2y+4)dy = 84 \qquad n > 0$$

Find the value of n. You must show full workings.

(2) Show, without a calculator, that

$$\int_3^{12} \left(\frac{1}{2\sqrt{p}} + \frac{3}{2}\sqrt{p}\right) dp = k\sqrt{3}$$

Where k is a constant to be found.

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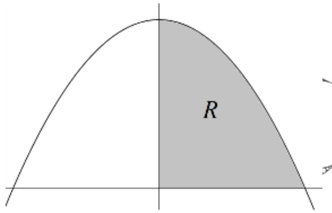
A LEVEL MATHS

(77) Integration (Basic Areas Under Curves)

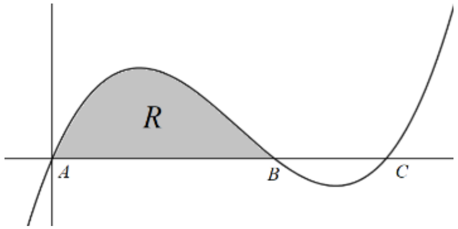
WORKING AT D/E

(1) The diagram below shows part of the curve with equation $y = 9 - x^2$



(a) Write down where the graph cuts the x axis.
The shaded region R is bounded by the curve with equation $y = 9 - x^2$, the positive x axis and the positive y axis as shown above.
(b) Use integration to show that the area of the region R is 18.

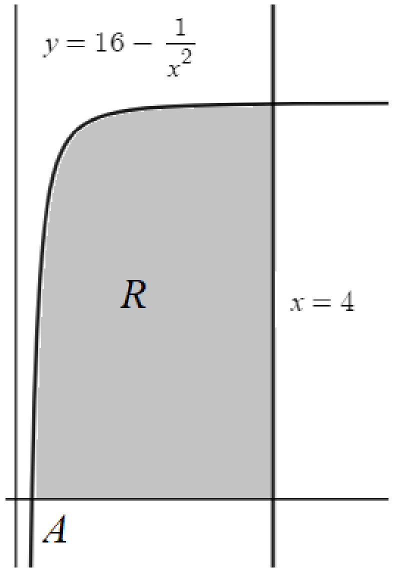
(2) (a) Factorise $x^3 - 5x^2 + 6x$ fully.
Part of the graph of $y = x^3 - 5x^2 + 6x$ is shown below. A, B and C are the points where the graph crosses the x axis.



(b) Write down the coordinates of A, B and C
(c) Use calculus to find the area of the shaded region R bounded between the curve and the x axis.

WORKING AT B/C

(1) The diagram below shows part of the curve with equation $y = 16 - \frac{1}{x^2}$, $x > 0$ and the line with equation $x = 4$.



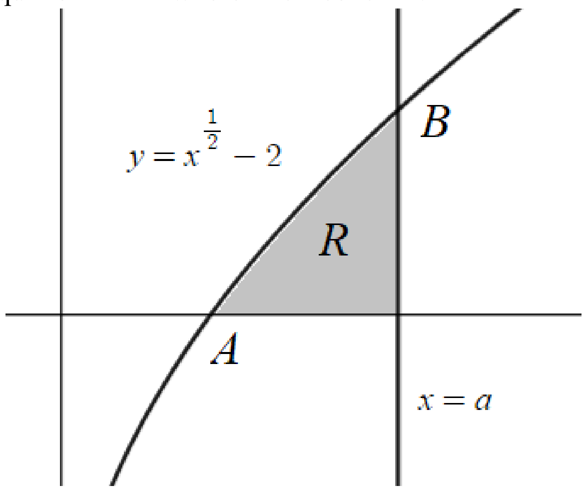
The graph of $y = 16 - \frac{1}{x^2}$ cuts the x axis at A.
(a) Find the coordinates of A.
The region R is bounded by the curve with equation $y = 16 - \frac{1}{x^2}$, the x axis and the line $x = 4$.
(b) Use calculus to show that the area of R is $\frac{225}{4}$

(2) (a) Sketch the curve of $y = x(4 - x)$
(b) Use calculus to find the **area** trapped between the curve and the positive x axis.

WORKING AT A*/A

(1) (a) Express $(x^2 - 1)(x^2 - 4)$ in the form $(x + a)(x + b)(x + c)(x + d)$
(b) Hence, sketch the graph of $y = (x^2 - 1)(x^2 - 4)$ showing the coordinates of the points where the graph crosses the coordinate axes.
(c) Find the area of the region trapped between the curve, the x axes and the lines $x = -1$ and $x = 1$

(2) The graph below shows part of the curve with equation $y = x^{\frac{1}{2}} - 2$, $x \geq 0$ and the line with equation $x = a$ where a is a constant.



The curve crosses the x axis at the point A.
(a) Find the coordinates of A
The line and the curve meet at the point B.
(b) Given that the coordinates of B are $(a, 1)$, find the value of a.
The region R is trapped between the x axis, the curve with equation $y = x^{\frac{1}{2}} - 2$ and the line $x = a$.
(c) Find the exact area of the region R.

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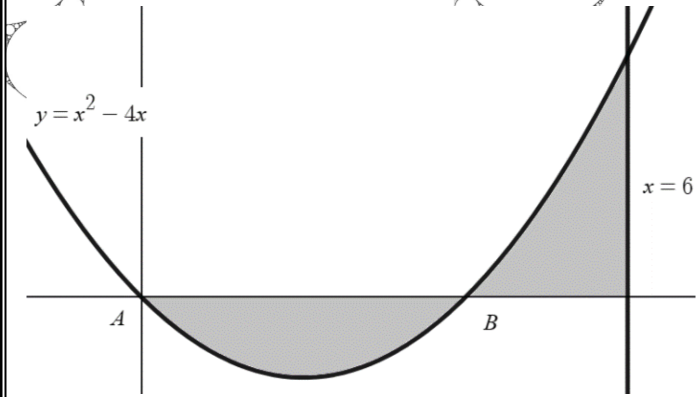
(78) Integration ('Negative and Positive Areas')

WORKING AT D/E

(1) The diagram below shows part of the curve with equation $y = x^2 - 4x$ and the line with equation $x = 6$



The curve crosses the x axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find $\int (x^2 - 4x)dx$
(c) Hence, using calculus, show that the total shaded area trapped between the curve, the positive x axis and the line $x = 6$ is $\frac{64}{3}$ units.

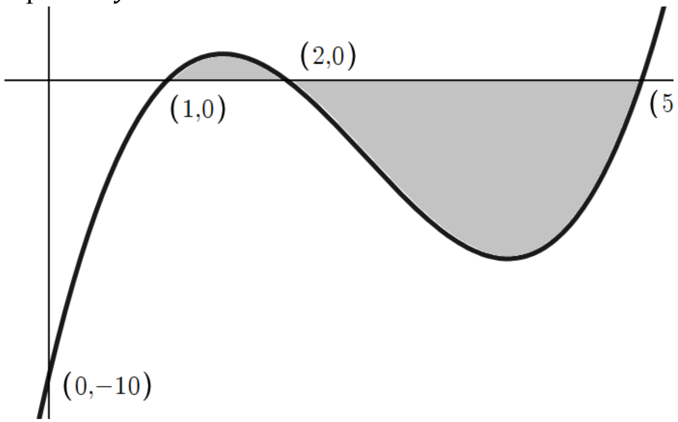
(2) (a) Sketch the curve of
$$y = (x+2)(x-1)(x-4)$$
(b) Hence, show without a calculator, that the total area trapped between the curve and the x axis from $x = -2$ to $x = 4$ is $\frac{81}{2}$ units.

WORKING AT B/C

(1) (a) Sketch the graph of $y = x^2 - x - 6$

(b) Hence, show that the total area trapped between the curve, and the x axis from $x = -2$ to the line with equation $x = 5$ is $\frac{67}{2}$

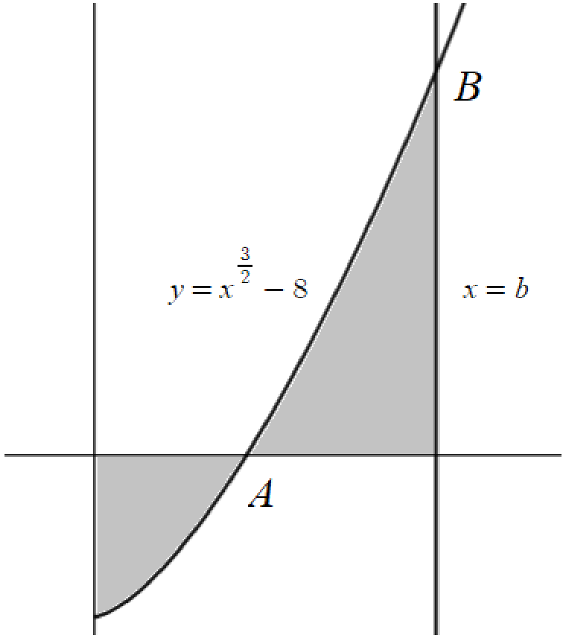
(2) The diagram below shows part of the graph with equation $y = x^3 + bx^2 + cx + d$



(a) Show that $b = -8,\ c = 17$ and $d = -10$
(b) Use integration to find the total shaded area.

WORKING AT A*/A

(1) The diagram below shows the curve of $y = x^{\frac{3}{2}} - 8,\ x \geq 0$ and the line with equation $x = b$



(a) The curve crosses the x axis at the point A. Find the coordinates of A.
(b) The curve and the line meet at the point B. Given that the coordinates of B are $(b, 19)$, find the value of b.
(c) Showing full workings, find the total shaded area shown trapped between the lines $x = 0$ and $x = b$ and the positive x axis.

(2) Showing full workings, find the total area trapped between the curve of $y = \frac{1}{x^2} - 3$, the positive x axes and the lines $x = \frac{1}{3}$ and $x = 1$. Give your answer in exact form.

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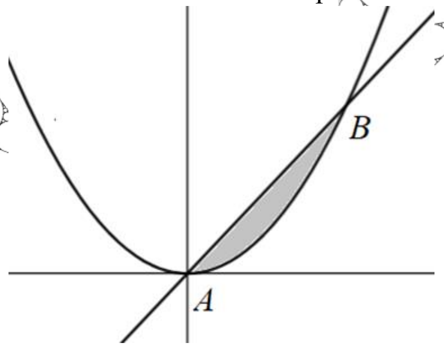
(79) Integration (Areas between Curves and Lines)

WORKING AT D/E

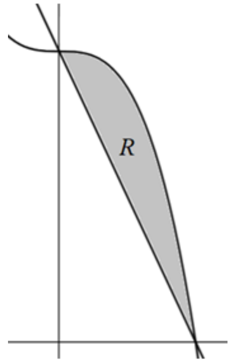
(1) The diagram below shows part of the curve with equation $y = x^2$ and the line with equation $y = 2x$. The line and curve intersect at the points A and B.



(a) Use simultaneous equations to find the coordinates of A and B.
The shaded area on the diagram is the region trapped between the line and the curve between the points A and B.
(b) Show, using calculus and using the area of a triangle, that the area of the shaded region is $\frac{4}{3}$

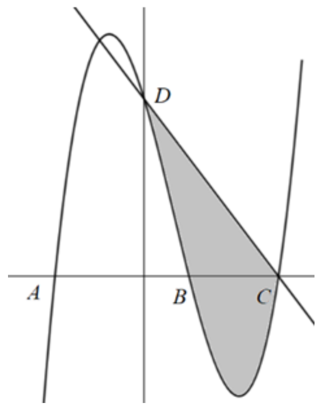
WORKING AT B/C

(1) The diagram below shows part of the curve with equation $y = -x^3 + 8$ and part of the line with equation $y = 8 - 4x$.



The region R is the area trapped between the curve and the line between where they intersect.
Use calculus to find the area of the shaded region R

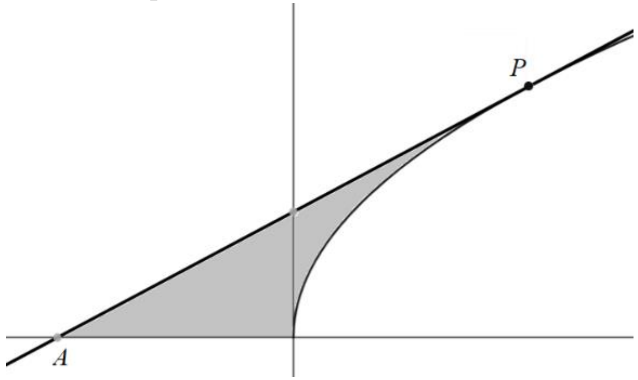
(2) The diagram below shows part of the curve with equation $y = (x-3)(x+2)(x-1)$ and the line with equation $y = 6 - 2x$. The line and curve intersect at the points C and D. The curve crosses the x axis at the points A, B and C.



Use calculus to show the shaded area is $\frac{45}{4}$

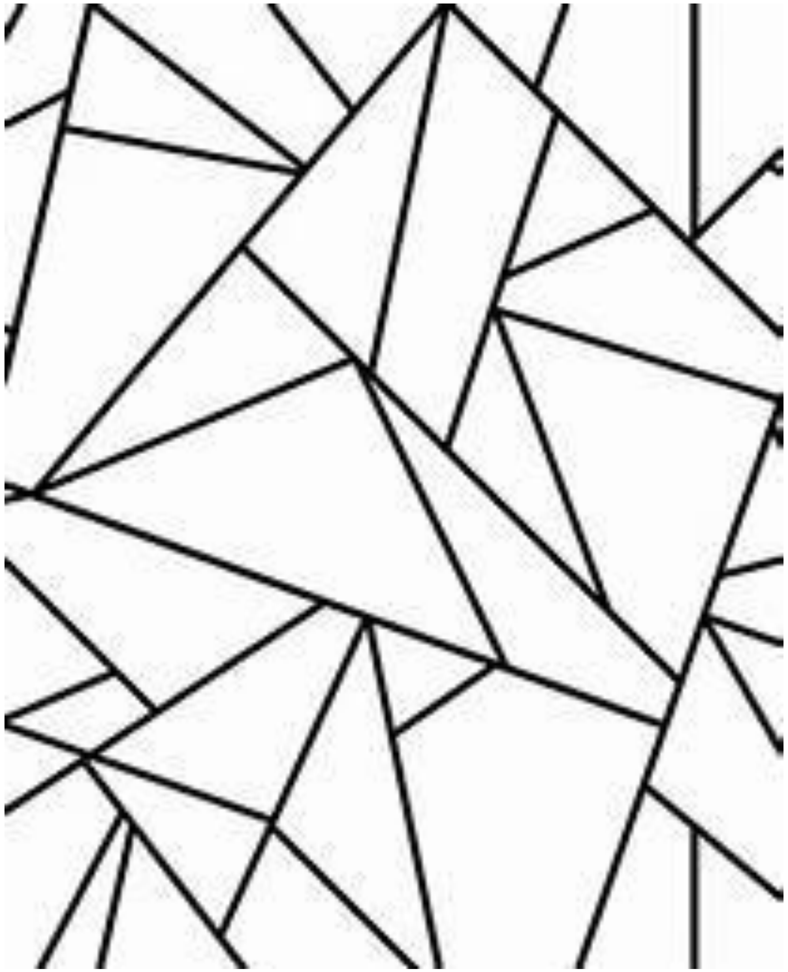
WORKING AT A*/A

(1) The diagram below shows part of the curve with equation $y = 4\sqrt{x},\ x \geq 0$ and the tangent to the curve at the point P.



The equation of the tangent is $y = x + a$ where a is a constant.
(a) Find the coordinates of P.
The tangent crosses the x axis at the point A.
(b) Find the coordinates of A
(c) Use calculus to show that the area of the shaded region shown above is $\frac{32}{3}$ square units.

Exponentials and Logarithms



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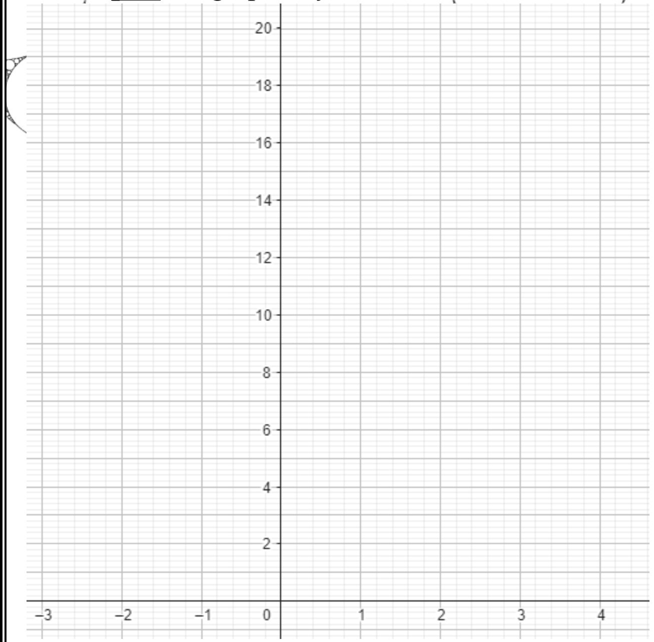
(80) Basic Exponential Functions

WORKING AT D/E

(1) (a) Using a set of axes like those in the diagram below, **plot** the graph of $y = 2^x, \ -3 \leq x \leq 4$



(b) Use the graph to estimate the value of $2^{2.5}$

(2) On the same set of axes sketch the graphs of $y = 2^x, \ \ y = 3^x$ and $y = 4^x$ showing where the graphs cross the coordinate axes.

(3) Sketch the graph of $y = \left(\frac{1}{2}\right)^x$

WORKING AT B/C

(1) $f(x) = 2^x$

(a) **Sketch** the graph of $y = f(x)$, showing where the graph crosses the coordinate axes and writing down the equation of the horizontal asymptote.

(b) On separate diagrams, sketch the following graphs:

(i) $y = 2f(x)$ showing where the graph crosses the y axis and stating the equation of the asymptote.

(ii) $y = f(x) + 3$ showing where the graph crosses the y axis and stating the equation of the asymptote.

(iii) $y = -f(x)$ showing where the graph crosses the y axis and stating the equation of the asymptote.

(iv) $y = f(-x)$ showing where the graph crosses the y axis and stating the equation of the asymptote.

(v) $y = f(x-1)$ stating the equation of the asymptote.

(2) The graph of $y = pa^x$ where p and a are constants passes through points (2,18) and (3,54)

(a) Show that $18 = pa^2$ and $54 = pa^3$

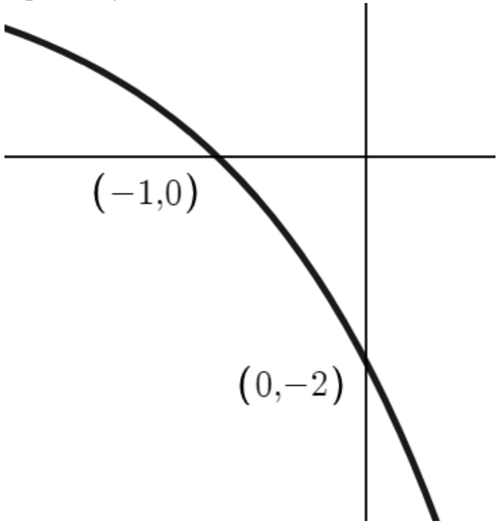
(b) Use simultaneous equations to find the values of p and a.

(c) Hence, **sketch** the graph of $y = pa^x$ showing where the graph crosses the y axis and stating the equation of the asymptote.

WORKING AT A*/A

(1) **Sketch** the graph of $y = 3\left(\frac{1}{2}\right)^{x-1}$ showing where the graph crosses the y axis and stating the equation of the asymptote.

(2) The diagram below shows part of the curve with equation $y = ab^x + 2$, where a and b are constants



The curve passes through the point $(3, p)$. Show that $p = -30$.

(3) The graph of $y = c + ab^x$, where a, b and c are constants, crosses the y axis at the point P
Find the coordinates of P in terms of a and c.

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(81) 'The' Exponential Function $y = e^x$

WORKING AT D/E

(1) **Sketch** the graph of $y = e^x$ showing where the graph crosses the y axis and stating the equation of the asymptote.

(2) Find the value of each, giving your answers to 2 decimal places:

(i) e^3 (ii) $e^{4.1}$ (iii) e^{-2}

(3) Find an expression for $\frac{dy}{dx}$ for each below:

(a) $y = e^x$ (b) $y = 3e^x$ (c) $y = e^{4x}$

(d) $y = e^x + x$ (e) $y = -e^x$ (d) $y = e^{-x}$

WORKING AT B/C

(1) (a) **Sketch** the graph of $y = 2e^x$ showing where the graph crosses the y axis and stating the equation of the asymptote.
(b) **Sketch** the graph of $y = 2-e^x$ showing where the graph crosses the y axis and stating the equation of the asymptote.
(c) **Sketch** the graph of $y = e^{x-3}$ showing where the graph crosses the y axis **in exact form** and stating the equation of the asymptote.

(2) (a) Find a simplified expression for $f'(x)$ for each below:

(i) $f(x) = e^{4x+1}$ (ii) $f(x) = e^x + x^2$

(iii) $f(x) = 4e^{3x}$ (iv) $f(x) = e^x(e^x - 6)$

(b) Given $f(x) = 2e^{5x}$, find $f'(2)$ giving your answer to 1 decimal place.

(3) Given that $y = (e^x + 1)^2$, show that

$$\frac{dy}{dx} = 2e^{2x} + 2e^x$$

WORKING AT A*/A

(1) A curve has equation $y = a + be^x$ where a and b are constants. Given that the point $(-1,\ 5 + \frac{2}{e})$
(a) Find the values of a and b.
(b) **Sketch** the graph of $y = a + be^x$ showing where the graph crosses the y axis and stating the equation of the asymptote.
(c) State the range of values that y can take.

(2) $f(x) = 7 - 5e^{x-2}$
The graph of $y = f(x)$ crosses the y axis at the point P.
(a) Write down the exact coordinates of P.
(b) The range of $f(x)$ is $f(x) < q$. Find the value of q.
(c) Find an expression for $f'(x)$.
(d) Hence, find the gradient of the curve when $x = 3$ giving your answer in exact form.

(3) $y = e^{3x}$
The normal to the curve at the point with x coordinate 1, crosses the coordinate axes at the points A and B.
Find coordinates for A and B giving your answers as exact values.

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(82) Applications of Basic Exponential Models

WORKING AT D/E

(1) Alan is growing a new colony of micro rats in an experiment. The number of rats N after time t weeks from the start of the experiment can be modelled by the equation $N = 10e^{0.2t}$

(a) Write down the initial number of rats at the start of the trial.
(b) Find the number of rats after 20 weeks.
(c) Show that $\frac{dN}{dt} = 2e^{0.2t}$
(d) Find the value of $\frac{dN}{dt}$ when $t = 8$.
(e) Interpret this value in the context of the model.
(f) Sketch the graph of $N = 10e^{0.2t}$ for $t \geq 0$
(g) State a limitation of the model.

WORKING AT B/C

(1) The number of people P after on a newly found island after n years can be modelled by the equation:

$$P = 40e^{0.1n} + 160, \quad n \geq 0$$

(a) Show that there were initially 200 people on the island.
(b) Find the number of people on the island after 12 years.
(c) Show that $\frac{dP}{dn}$ can be written in the form $ke^{0.1n}$ where k is an integer.
(d) What does $\frac{dP}{dn}$ represent in the context of the model?
(e) Find the value of $\frac{dP}{dn}$ when $n = 20$
(f) Sketch the graph of $P = 40e^{0.1n} + 160$

(2) The amount of moss observed on a rock Mkg after time t years can be modelled by the equation

$$M = 2 + 3e^{-\frac{t}{8}}, \ t \geq 0$$

(a) Find the amount of moss initially observed.
(b) Does the equation model growth or decay? You must justify your answer.
(c) Find the amount of moss on the rock after 12 years. Give your answer to the nearest 100g.
(d) Show that $\frac{dM}{dt} = -0.375e^{-\frac{t}{8}}$
(e) Find $\frac{dM}{dt}$ when $t = 9$
(f) Interpret this value in context of the model
(g) Beryl believes there will always be at least 1kg of moss on the rock. Is she correct? You must justify your answer.
(h) Sketch the graph of $M = 2 + 3e^{-\frac{t}{8}}, \ t \geq 0$

WORKING AT A*/A

(1) The value of a boat V £ after t years can be modelled by the equation $V = 8000 + \frac{12000}{e^{\frac{1}{4}t}}, t \geq 0$

(a) Explain why this equation models depreciation.
(b) Find the initial value of the boat.
(c) Find the value of the boat after 8 years giving your answer to the nearest £.
(d) Find the rate at which the boat is depreciating after 10 years.
(e) Sketch the graph of V against t.
(f) Interpret the asymptote on the graph in context of the model.
(g) Make one criticism of the model.

(2) The population of a newly inhabited island can be modelled by the equation $P = 100 + Ae^{bt}$
Where P is the number of people (in thousands) and n is the number of years after the island was first inhabited. A and b are constants.
(a) Given that there were initially 120'000 people on the island, find the value of A.
The rate at which the population is increasing after n years can be found using the expression $6e^{bt}$
(b) Find the value of b.
(c) Find the population after 10 years.
(d) Sketch the graph of P against t.
(e) Use logarithms to find the rate at which the population is increasing at a rate of 40000 people a year.

(3) A model has the equation $A = b + ce^{dt}$ where b, c and d are positive constants and t is time. Find a general expression for the rate at which A is changing.

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A LEVEL MATHS

(83) Logarithms (Simplifying & Evaluating)

WORKING AT D/E

(1) Rewrite each of the following using a logarithm.

(a) $3^2 = 9$ (b) $5^3 = 125$ (c) $8^2 = 64$

(d) $4^{-1} = \frac{1}{4}$ (e) $9^0 = 1$ (f) $8^{\frac{2}{3}} = 4$

(2) Without a calculator, find the value of each:

(a) $\log_2 8$ (b) $\log_3 81$ (c) $\log_4 16$

(d) $\log_5 125$ (e) $\log_2 32$ (f) $\log_7 7$

(3) Use your calculator to find the value of each to 3SF.

(a) $\log_2 27$ (b) $\log_7 3$ (c) $\log_{0.1} 0.05$

WORKING AT B/C

(1) Without a calculator, find the value of x in each:

(a) $\log_2 x = 3$ (b) $\log_3 1 = x$ (c) $\log_4 2 = x$

(d) $\log_3 3 = x$ (e) $\log_6\left(\frac{1}{36}\right) = x$ (f) $\log_5 0.2 = x$

(g) $\log_2(x-1) = 4$ (h) $\log_5(2x) = 4$

(2) Given that $\log x$ is the same as $\log_{10} x$, without a calculator, find the value of each.

(a) $\log 100$ (b) $\log 0.1$ (c) $\log 1$

(3) Explain why $\log_a a^b = b$ for when a is positive and $a \neq 1$.

WORKING AT A*/A

(1) Given that $x > 0$, without a calculator, find the value of x in each:

(a) $\log_x 9 = 2$ (b) $\log_4(3-x) = 1$

(c) $\log_5 0.04 = x - 3$ (d) $\log_4 1 = 2x - 1$

(e) $\log_x 0.125 = -3$ (f) $\log_8 2 = x + 7$

(2) Without using a calculator, **estimate** the value of x in each:

(a) $\log_3 25 = x$ (b) $\log_4 14 = x$

(c) $\log_2 x = 3.5$ (b) $\log 110 = x$

(3) Alan is trying to solve the inequality below for x.

$$(\log_8 0.5)\, x > 14$$

He writes:

$$x > \frac{14}{(\log_8 0.5)}$$

$$x > -42$$

Is he correct? You must justify your answer.

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(84) Logarithms (The Log Laws)

WORKING AT D/E

(1) Write each of the following as a single logarithm:

(a) $\log 2 + \log 8$ (b) $\log 15 - \log 5$

(c) $\log 3 + \log 8 - \log 2$ (d) $\log 2 - \log 5$

(2) (a) Show that $\log x^2 y^3 \equiv 2\log x + 3\log y$

(b) Show that $\log \frac{x^5}{\sqrt{y}} \equiv 5\log x - \frac{1}{2}\log y$

(3) (a) By combining logarithms show that the equation

$$\log_2 3 + \log_2(x-1) = 0$$

can be written as $3(x-1) = 1$.

(b) Hence, solve the equation

$$\log_2 3 + \log_2(x-1) = 0$$

WORKING AT B/C

(1) Write each of the following in terms of $\log_2 x$, $\log_2 y$ and $\log_2 z$.

(a) $\log_2\left(\frac{x^6}{y}\right)$ (b) $\log_2 x^7 z y^3$ (c) $\log_2 8xz^3$

(2) Solve the equation

$$\log_2(5x-6) + \log_2(3x+10) = 6$$

Giving your answer as an integer.

(3) (a) Show that the equation

$2\log_3(2x+1) = 5 - \log_3(x-1)$,

Can be written as $(2x+1)^2(x-1) = 243$

(b) Hence, **verify** the solution $x = 4$ is a solution to the equation $2\log_3(2x+1) = 5 - \log_3(x-1)$

WORKING AT A*/A

(1) (a) Find the solution to the equation

$2\log_4(x-1) = 0.5 + \log_4(x+3), \quad x \in R$

Showing step by step workings.

(b) Explain why there is only one solution to the equation.

(2) Beryl is trying to find the real solutions to the equation

$$a\log_b(4x+3) = c, \quad x \in R$$

Find the set of values for which x is valid.

(3) (a) Given that $p = \log_8 x$ and $q = \log_8 y$, write each of the following in terms of p and q

(i) $\log_8 2x^4 y^{\frac{1}{3}}$

(ii) $\log_8 \frac{x^9}{4\sqrt{y}}$

(b) Write the following as a single logarithm

$$100 + 2\log x - 0.5\log y$$

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(85) Logarithms (Log and Exponential Equations)

WORKING AT D/E

(1) Solve each equation giving your answer to 3.SF
(a) $3^x = 13$ (b) $5^x = 16$ (c) $2^x = 0.91$

(2) Solve each equation giving your answer to 3.SF
(a) $4^{x+1} = 50$ (b) $5^{1-x} = 8$ (c) $7^{2x+1} = 100$

(3) (a) By taking logarithms of both sides of the equation, show that the equation $5^x = 2^{3x-1}$ can be written as $x \log 5 = 3x \log 2 - \log 2$

(b) Hence, solve the equation $5^x = 2^{3x-1}$ giving your answer to 3 significant figures.

WORKING AT B/C

(1) Solve the equation $4^{x+1} = 3^{2x-1}$ giving your answer to 3 significant figures.

(2) (a) Using the substitute $p = 5^x$ show that the equation $5^{2x} + 2(5^x) - 8 = 0$ can be written as $(p + a)(p + b) = 0$ where a and b are constants to be found.

(b) Hence, find the solutions for p.

(c) Using your answer to part (b) find the only solution to the equation $5^{2x} + 2(5^x) - 8 = 0$.

(3) Find the coordinates of the point where the graphs of $y = 2^x$ and $y = 3^{x-1}$ intersect giving your answers to 3 significant figures.

WORKING AT A*/A

(1) Solve the equation $2^{3x-1} = 7 \times 5^{x-3}$ giving your answer to 3 SF.

(2) Solve the equation $6^{2x} - 6^{x+1} + 8 = 0$

(3) Prove that there are no real solutions to the equation $5(2^{2x+1}) + 2^{x+2} + 1 = 0$

Thank you