Complex Number Numbers – CP2 Edexcel – Steve Blades – www.m4ths.com

(1) Show that $\left(3-i\sqrt{3}\right)^{12}$ is a real number that can be written in the from a^n.

(2) (a) Show that $\sin^4\theta$ can be written in the form $A\cos 4\theta + B\cos 2\theta + C$ where A, B and C are constants to be found.
(b) Hence, show that:

$$32\int_0^{\frac{\pi}{4}} \sin^4\theta = 3\pi - 8$$

(3) $z = \frac{\sqrt{3}+i}{\sqrt{2}(1+i)}$
(a) Write z in the mod/arg form.
(b) Plot z on an Argand diagram.
(c) Find the coordinates of z in Cartesian form.
(d) The point P has the coordinates found in part (c) of the question. P is one of the vertices of an equilateral triangle. Find the coordinates other two vertices of the equilateral triangle.

(4) Solve the equation $z^5 - 4 + 4i = 0$ giving your answers in exponential form.

(5) z and w are 2 different complex numbers such that:

$$|z| = 4$$
$$\arg\left(\frac{w}{z}\right) = \frac{\pi}{6}$$
$$w = -4 + 4\sqrt{3}i$$

Show that z is purely imaginary and plot it on an Argand diagram.

(6) Given that $z = e^{\frac{\pi i}{4n}}$, show that:

$$1 + z + z^2 + z^3 + \cdots + z^{8n-1} = 0$$

(7) The coordinates of one vertex a square are $\left(1, -\sqrt{3}\right)$.
(a) Find the coordinates of the other 3 vertices.
(b) Find the area of the square.
(c) The midpoints to the sides of the square are the $4th$ roots of another complex number w.
Find a possible expression for w in the form $re^{i\theta}$.

(8) Find the $6th$ roots of unity giving your answers in the form $x + iy$.

(9) Two convergent infinite series C and S are given as:

$$C = 1 + \frac{1}{5}\cos\theta + \frac{1}{25}\cos 2\theta + \frac{1}{125}\cos 3\theta + \cdots$$
$$S = \frac{1}{5}\sin\theta + \frac{1}{25}\sin 2\theta + \frac{1}{125}\sin 3\theta + \cdots$$

(a) Show that:

$$C + iS = \frac{5}{5 - e^{i\vartheta}}$$

(b) Hence show that:

$$C = \frac{25 - 5\cos\theta}{26 - 10\cos\theta}$$

(c) Find a similar expression for S.

(10) (a) Show that $\cos 5\theta = \cos\theta(\cos^4\theta - 10\cos^2\theta\sin^2\theta + 5\sin^4\theta)$
(b) Hence, solve the equation $\cos\theta(\cos^4\theta - 10\cos^2\theta\sin^2\theta + 5\sin^4\theta) = \sec 5\theta,\ 0 \le \vartheta \le \frac{\pi}{2}$
(c) Prove by induction that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$