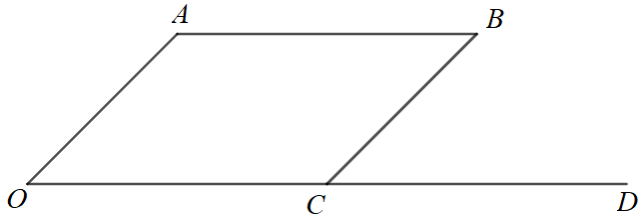
Vectors in Geometry – wwww.m4ths.com

(1) The diagram below shows the parallelogram $OABC$.



$\overrightarrow{OA} = \text{a}$, $\overrightarrow{AB} = \text{b}$ and $OC = CD$

X lies on OA such that $OX : XA$ is 2: 1 and Y is the midpoint of AB.

(a) Find the following vectors in terms of a and b fully simplifying your answers.

(i) $\overrightarrow{OB}$ (ii) $\overrightarrow{AO}$ (iii) $\overrightarrow{OC}$ (iv) $\overrightarrow{BC}$ (v) $\overrightarrow{OD}$ (vi) $\overrightarrow{BD}$ (vi) $\overrightarrow{BA}$ (vii) $\overrightarrow{CO}$

(b) Find the following vectors in terms of a and b fully simplifying your answers.

(i) $\overrightarrow{OX}$ (ii) $\overrightarrow{AY}$ (iii) $\overrightarrow{OY}$ (iv) $\overrightarrow{BX}$ (v) $\overrightarrow{XY}$ (vi) $\overrightarrow{YX}$

(c) Prove that $\overrightarrow{XC}$ is parallel to $\overrightarrow{YD}$

The point Z is the midpoint of BC

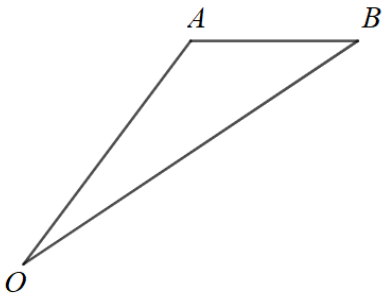
(d) Prove that AZD is a straight line.

(e) Explain why the points A, Z and D are collinear.

N is the midpoint of CD

(f) Show that $\overrightarrow{YC}$ and $\overrightarrow{NB}$ are the same vector.

(2) The diagram below shows the triangle OAB.



$\overrightarrow{OA} = 3\text{p} + 4\text{q}$ and $\overrightarrow{OB} = 6\text{p} + 4\text{q}$

(a) Show that $\overrightarrow{AB}$ can be written in the form kp.

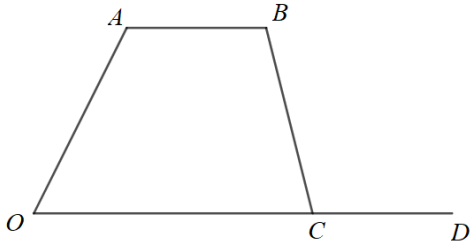
The point X is the midpoint of OB.

(b) Show that $\overrightarrow{OA}$ can be written in the form nq.

The point Y is such that $\overrightarrow{OY} = 2\overrightarrow{AB}$

(c) Show that $\overrightarrow{BY} + \overrightarrow{AY}$

(3) The diagram below shows that trapezium $OABC$.



$\overrightarrow{OA} = \text{a}$, $\overrightarrow{AB} = \text{b}$, $AB = CD$ and the ratio $OC : CD$ is 2: 1

(a) Find the following vectors in terms of a and b fully simplifying your answers.

(i) $\overrightarrow{OB}$ (ii) $\overrightarrow{AO}$ (iii) $\overrightarrow{OC}$ (iv) $\overrightarrow{BC}$ (v) $\overrightarrow{OD}$ (vi) $\overrightarrow{BD}$ (vi) $\overrightarrow{BA}$ (vii) $\overrightarrow{CO}$

The point X is the midpoint of OC.

(b) Show that $\overrightarrow{OA}$ and $\overrightarrow{BX}$ are a parallel.

The point Y lies on the line OC such that $OY : YC$ is 5: 1

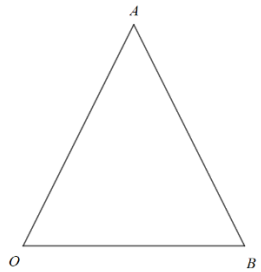
(c) Prove that $\overrightarrow{AY}$ and $\overrightarrow{BD}$ are a parallel.

The point P is the midpoint of OA and the point Q is the midpoint of BC.

(d) Show that Show that $\overrightarrow{OD}$ and $\overrightarrow{PQ}$ are a parallel.

(e) Find the ratio $PQ : OD$

(4) The diagram below shows the triangle OAB.



$\overrightarrow{OA} = 4\text{p} + 8\text{q}$, $\overrightarrow{OB} = 8\text{p}$

The point X is the midpoint of OB.
The point Y lies on OB such that $OY : YB$ is 3: 1
The point Z lies on OA such that $OZ : ZA$ is 3: 1
The point T is the midpoint AX

(a) Prove that ZTY is a straight line

(b) Find the ratio $ZT : TY$