Methods in Differential Equations – Chapter 7 Test - CP2 – www.m4ths.com – Steve Blades!

(1) (a) Show that the general solution to the differential equation $\frac{dy}{dx} + 3y = xe^{-x}$ can be written in the form $y = \frac{1}{4}e^{-x}(2x - 1) + ce^{-3x}$ where c is a constant.
(b) Find the particular solution to the equation given that when $x = 0, y = 3$
(c) Describe the behaviour of the function for large values of x.

(2) Find the particular solution to the differential equation $\frac{d^2y}{dx^2} + 4y = 6\sin x$
Given that when $x = 0,\ y = 2$ and $\frac{dy}{dx} = 1$.

(3) Show that the general solution to the differential equation $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} = 2x + 4$
is $y = A + Be^{8x} - \frac{1}{8}x^2 - \frac{17}{32}x$ where A and B are constants.

(4) Find the particular solution to the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$ given that when $x = 0$, $y = 6$ and $\frac{dy}{dx} = -12$.

(5) Find the general solution to the differential equation $\cot x\frac{dy}{dx} - y\mathrm{cosec}^2\, x = \sec^2 x$ giving your answer in the form $y = f(x)$.

(6) Prove that the solution to the differential equation $\frac{dy}{dx} + yx - x = 0$ is an exponential function.

(7) Find the particular solution to the differential equation $\frac{dy}{dx} - \frac{y}{1+x} = e^x(1 + x)$ given that when $y = 5$ when $x = 0$ giving your answer in the form $y = f(x)$.

(8) Find the general solution to the differential equation $\frac{d^2x}{dt^2} + \frac{dx}{dt} - 12x = 14e^{3t}$

(9) The general solution to the differential $\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0$ is $y = e^{-4x}(A\cos 7x + B\sin 7x)$ where a, b, A and b are constants.
(a) Find the value of a and the value of b.
(b) Explain why you can't find the value of A and B.

(10) (a) Find the particular solution to the differential equation $x\frac{dy}{dx} + \frac{y}{\ln x} = \frac{x}{\ln x},\ x > 1$ given that when $x = e, y = 1$ giving your answer in the form $y = f(x)$.
(b) Describe the behaviour of $f(x)$ for large values of x giving a justification for your answer.

(11) (a) Find the general solution to the differential equation:

$$\frac{d^2y}{dx^2} = -y$$

(b) **Write down** the form of the particular integral required to solve the differential equation:

$$\frac{d^2y}{dx^2} = -y + 2\sin x$$