Sine and Cosine www.m4ths.com

(1) $p^2 = \sqrt{8^2+13^2-2(8)(13)\cos(97)}$

$p = 16.1$

$\cos^{-1}\left(\dfrac{6.7^2+6.2^2-7.1^2}{2(6.7)(6.2)}\right) = q$

$q = 66.7^\circ$

$\dfrac{r}{\sin 53} = \dfrac{2.9}{\sin 41}$

$r = 3.53$

$\dfrac{\sin(S)}{12.7} = \dfrac{\sin(67)}{19.1}$

$S = 37.7^\circ$

(2) $\dfrac{\sin x}{5.27} = \dfrac{\sin 80}{7.21}$

$x = 46.04$

$\therefore p = 180 - x - 80$

$p = 53.9598\ldots$

$p = 54.0^\circ$

$\dfrac{q}{\sin 28} = \dfrac{1.8}{\sin 55}$

$q = 1.03$

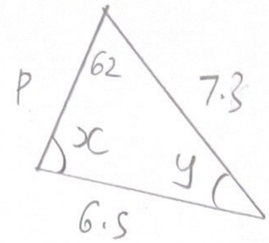
$r = \cos^{-1}\left(\dfrac{12^2+16.5^2-7.9^2}{2(12)(16.5)}\right)$

$r = 26.7^\circ$

$S = \sqrt{6.3^2+8.4^2-2(6.3)(8.4)\cos(92)}$

$S = 10.7$

(3)



$\dfrac{\sin x}{7.3} = \dfrac{\sin 62}{6.5}$

$x = 82.576\ldots$

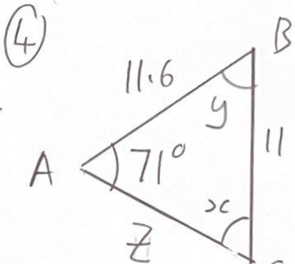
$\therefore y = 180 - 62 - 82.576\ldots$

$y = 35.423$

$\dfrac{P}{\sin y} = \dfrac{6.5}{\sin 62}$

$P = 4.27$

(4)



$\dfrac{\sin x}{11.6} = \dfrac{\sin 71}{11}$

$\sin x = 0.99709$

$x = \sin^{-1}(0.9970\ldots)$

(a) $x = 85.62969.$

or $x = 180 - 85.6296.$

$x = 94.3703$

(b) $x = 94.37\ldots$

$y = 180 - 94.37 - 71 = 14.629$

$z^2 = 11.6^2 + 11^2 - 2(11.6)(11)\cos y$

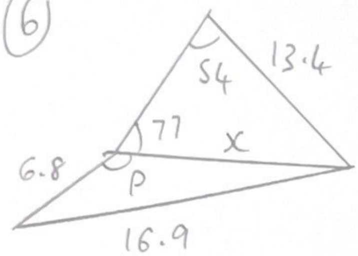
$z = 2.94$ (z is AC).

(5) $\cos p = \dfrac{12^2+15^2-10^2}{2(12)(15)}$

$= \dfrac{144+225-100}{360}$

$= \dfrac{269}{360}$

(6)



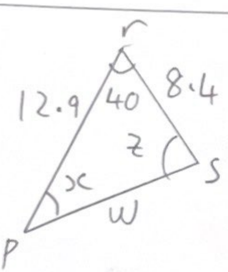
$$\frac{x}{\sin 54} = \frac{13.4}{\sin 77}$$

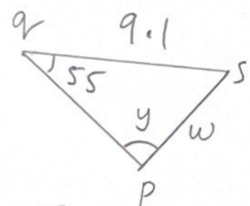
$x = 11.125..$

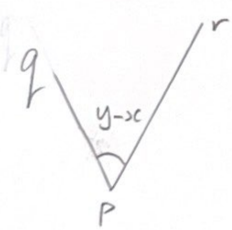
$$\cos P = \frac{6.8^2 + 11.125..^2 - 16.9^2}{2(6.8)(11.125..)}$$

$P = 139.8$ to 1dp.

(7)







Angle $rpq = y - x$

$W = \sqrt{12.9^2 + 8.4^2 - 2(12.9)(8.4)\cos 40}$

$W = 8.4233513 69$

$$\frac{\sin x}{8.4} = \frac{\sin 40}{w}$$

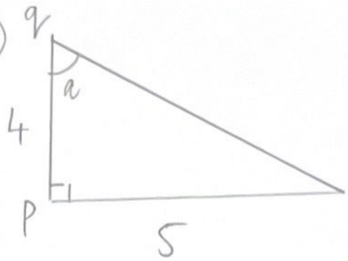
$x = 39.86685..$

$$\frac{\sin y}{9.1} = \frac{\sin 55}{w}$$

$y = 62.2459..$

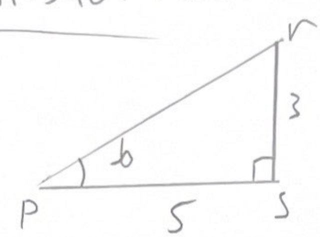
$\therefore \angle rpq = y - x = 22.4$ to 3SF

(8)



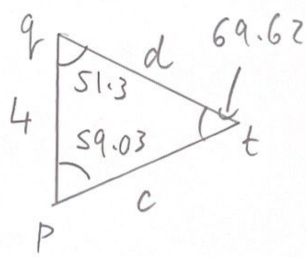
$a = \tan^{-1}\left(\frac{5}{4}\right)$

$a = 51.340 19...$



$b = \tan^{-1}\left(\frac{3}{5}\right)$

$b = 30.9637..$



$\angle qpt = 90 - b$

$\angle qtp = 180 - 51.3.. - 59.$

$= 69.62...$

$$\frac{c}{\sin(51.3)} = \frac{4}{\sin(69.62)}$$

$c = 3.33197...$

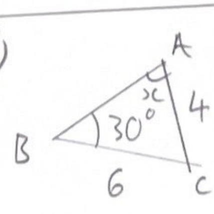
$$\frac{d}{\sin(59.03)} = \frac{4}{\sin(69.62)}$$

$d = 3.65877...$

$\therefore$ perimeter $= 4 + 3.33 + 3.65..$

$= 11.0$ to 3SF

(9)



$$\frac{\sin x}{6} = \frac{\sin 30}{4}$$

$$\sin x = \frac{6 \times \frac{1}{2}}{4}$$

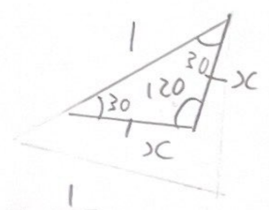
$= 3/4$ or 0.75

(10)

$p = \sqrt{17.5^2 + 13.5^2 - 2(17.5)(13.5)\cos(92.5)}$

$p = 22.563$ to 3dp.

(11) The polygon is a regular Hexagon ∴ each interior Angle is 120°.
The triangle is equilateral ∴ each length is 1 unit.



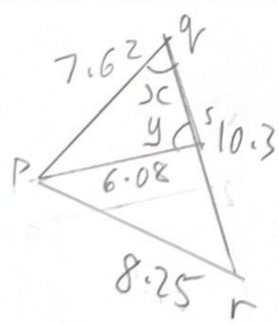
$\frac{x}{\sin 30} = \frac{1}{\sin 120}$

$x = \frac{\sqrt{3}}{3}$

Perimeter of hexagon $= 6x$

$\therefore 6\left(\frac{\sqrt{3}}{3}\right) = 2\sqrt{3}$ ✓

(12)



$\cos x = \frac{7.62^2 + 10.3^2 - 8.25^2}{2(7.62)(10.3)}$

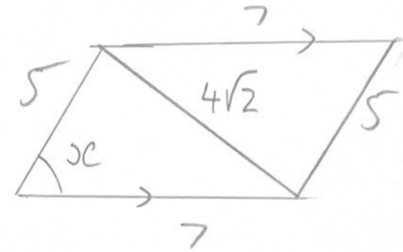
$x = 52.254$

$\frac{\sin y}{7.26} = \frac{\sin x}{6.08}$

$y = \sin^{-1}\left(\frac{7.26\sin(52.254)}{6.08}\right)$

$y = 70.8°$ $\therefore \angle qsp = 70.8°$

(13)



Smallest angle $= x$

Largest angle $= 180 - x$

$\cos x = \frac{5^2 + 7^2 - (4\sqrt{2})^2}{2(5)(7)}$

$x = 53.13$

$\therefore y = 126.9°$ to 1dp.

(14)

(a) $\frac{p}{\sin P} = \frac{q}{\sin Q}$

$\frac{p}{\frac{1}{3}} = \frac{q}{\frac{1}{5}}$

$p = \frac{5}{3}q$

(b) Triangle Law $p + q > r$ as $\angle P > 0$ and $\angle Q > 0$

$\therefore q + \frac{5}{3}q > r$

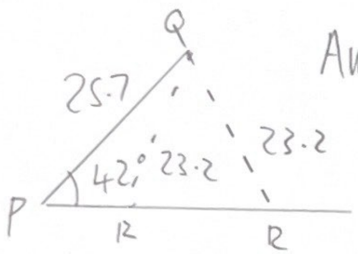
$\frac{8}{3}q > r$ ✓

(c) Many possible ways:
Example: $\sin 0/\sin 180 = 0$ and $\sin 30 = \frac{1}{2}$
$\frac{1}{3}$ and 0.2 are less than 0.5 they are either less than 30° or greater than 150°. As $q < p < r$, $\angle R$ must be the largest angle.

(c) Cont

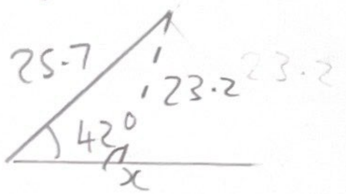
You can only have one obtuse angle in a triangle
$\therefore$ $\angle P$ and $\angle Q$ are acute and $\angle R$ is obtuse.

(15)



Ambiguous Case

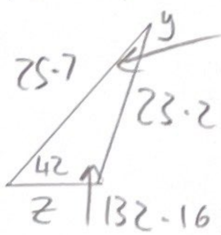
Taking the smaller triangle.



$$\frac{\sin x}{25.7} = \frac{\sin 42}{23.2}$$

$\sin x = 0.741\ldots$

$\therefore x = 47.836$ or $132.16\ldots$



$180 - 132.16 - 42$

$y = 5.836$

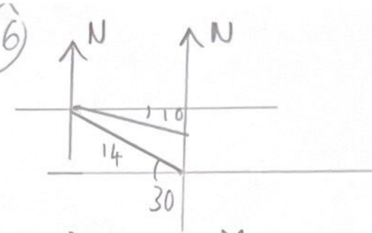
Sine Rule (or Cosine)

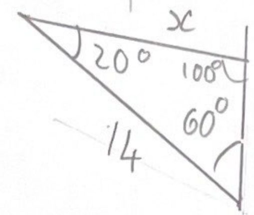
$$\frac{z}{\sin y} = \frac{23.2}{\sin 42}$$

$z = 3.5292\ldots$

$\therefore$ Perimeter $= 25.7 + 23.2 + 3.5\ldots$
$= 52.4$ to 3SF cm

(16)





$$\frac{x}{\sin 60} = \frac{14}{\sin 100}$$

$x = 12.311\ldots$

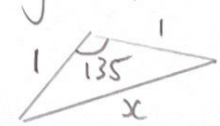
Total distance: $14 + 12.311 = 26.311$

Time

$$\frac{26.3113}{5} = 5.2622 \text{ hrs}$$

which is 5hrs 15mins 44secs
$\therefore$ 5:16pm.

(17) length of dotted line.

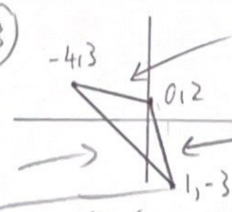


$x = \sqrt{1^2 + 1^2 - 2(1)(1)\cos 135}$

$x = 1.847759065$

Total length $= 6(1) + 7(1.8477\ldots)$
$= 6 + 12.934\ldots$
$= 18.9$cm to 3SF.

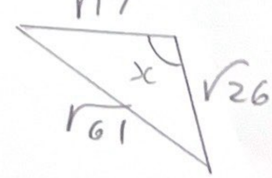
(18)



$\sqrt{(-4-0)^2 + (3-2)^2} = \sqrt{17}$

$\sqrt{(1-0)^2 + (-3-2)^2}$

$\sqrt{(-4-1)^2 + (3--3)^2}$
$= \sqrt{61}$



(a)

(18) Cont

$$\cos x = \frac{(\sqrt{17})^2 + (\sqrt{26})^2 - (\sqrt{61})^2}{2(\sqrt{17})(\sqrt{26})}$$

$$= \frac{17+26-61}{2\sqrt{442}}$$

$$= \frac{-9}{\sqrt{442}}$$

$$= \frac{-9\sqrt{442}}{442}$$

(b) $\cos^2 x + \sin^2 x \equiv 1$

$\therefore \sin x = \sqrt{1-\cos^2 x}$

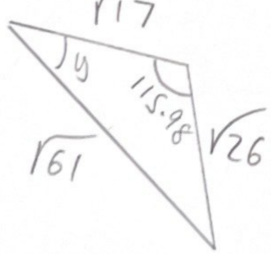
$\sin x = \sqrt{1 - \left(\frac{-9}{\sqrt{442}}\right)^2}$

$\sin x = 0.8989198363$

using a calculator.

(c) Method 1

Using Trig



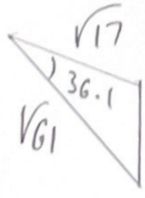
$$\cos y = \frac{17+61-26}{2(\sqrt{61})(\sqrt{17})}$$

$y = 36.158\ldots$

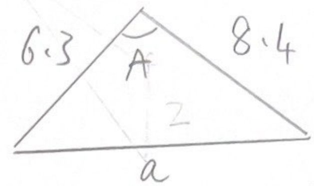
Area $A = \frac{1}{2}ab\sin C$

$A = \frac{1}{2}(\sqrt{17})(\sqrt{61}) \times \sin 36$

$= 9.5$



(19) Using the ambiguous case of the sine rule.



to maximise perimeter A must be the largest angle and a must be the longest side.

Area $\frac{1}{2}(6.3)(8.4)\sin A = 22.3$

$\sin A = \frac{1115}{1323}$

$\therefore A = 57.435$ or $A = 122.5649$

Cosine Rule

$a = \sqrt{6.3^2 + 8.4^2 - 2(6.3)(8.4)\cos(122.564)}$

$a = 12.931\ldots$

$\therefore$ Max perimeter $=$

$6.3 + 8.4 + 12.93 = 27.6\text{ cm}$