## Solving Simultaneous Equations Graphically

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## Starter - Locating Points

(1) Write down the coordinates of A, B, C, D \& E.
(2) Estimate the values of coordinates of F, G, H \& I.


Section 1 - Linear Simultaneous Equations
(1) Use the diagram below to find the_solutions of

$$
\begin{aligned}
& y=2 x-1 \\
& y=x+3
\end{aligned}
$$


(2) Use the 2 graphs below to solve the simultaneous equations given in the diagram.

(3) Use the diagram below to estimate the solutions to the simultaneous equations:

$$
y=5-2 x
$$

$$
y=x+1
$$


(4) Use the 2 graphs below to estimate the solutions of simultaneous equations given in the diagram.

(5) Extension: Can you solve Q1-4 using algebra?

## Section 2 - Real Life Applications

(1) Company A and Company B are two different taxi companies. The cost of a journey with each company is shown below in the diagram.

(a) Find the cost of a 2 mile journey with each company.
(b) Find the cost of a 5 mile journey with each company.
(c) Fred wants to travel home one day. Which company should Fred use? You must state clearly why you chose.
(2) The diagram below shows the revenue (money taken from sales) and productions costs (money spent on production) of making toys.


The break-even point is where the revenue $\&$ costs are equal. Use the graph to estimate the number of toys the company needs to break even and the cost of producing that number of toys.

## Section 3 - Quadratic Simultaneous Equations

(1) The diagram below shows part of the graphs of $y=x+4$ and $y=x^{2}+2 x-8$

(a) Find the coordinates of the points where the two graphs intersect (where the line crosses the curve).
(b) Using your answer to part (a) write down the solutions to the simultaneous equations:

$$
\begin{aligned}
& y=x+4 \\
& y=x^{2}+2 x-8
\end{aligned}
$$

The line $y=0$ is the $x$ axis.
(c) Use the graph to solve the equation
$0=x^{2}+2 x-8$
(2) The diagram below shows part of the graphs of $y=3$ and $y=x^{2}-x-6$.

(a) Use the diagram above to estimate the coordinates of both points where the two graphs intersect.
(b) Hence estimate the solutions to the equation $3=x^{2}-x-6$.
(c) Draw the line $y=-2$ on the graph above and hence estimate the solutions to the equation $x^{2}-x-6=-2$
(c) Hence write down the solutions to the simultaneous equations:

$$
y=-2
$$

$$
y=x^{2}-x-6
$$

## Section 4 - Drawing Graphs to Solve Equations

(1) (a) Using either a table of values or the gradient/intercept method, draw the graphs of $y=4 x-2$ and $y=2 x+3$ on the grid below.

(b) Use your graphs to estimate the solutions to the
simultaneous equations:

$$
\begin{aligned}
& y=4 x-2 \\
& y=2 x+3
\end{aligned}
$$

(2) (a) Draw the graphs of $y=4-x$ and $y=3 x-1$ on the grid below.

(b) Use your graphs to estimate the solutions to the
simultaneous equations:

$$
\begin{aligned}
& y=4 x-2 \\
& y=2 x+3
\end{aligned}
$$

(3) (a) By drawing the graph of $y=2$ on the grid below, find the solutions to the equation $2=4-2 x^{2}$

(b) Draw the line $y=x+1$ on the grid above and hence estimate the solutions to: $y=4-2 x^{2}$

$$
y=x+1
$$

(4) Extension: use algebra to solve all of the simultaneous equations given in this section.

## Section 5 - Extension (Sketching graphs may help)

(1) Write down the number of solutions of the
simultaneous equations:

$$
\begin{aligned}
& y=m x+3 \\
& y=m x-3
\end{aligned}
$$

(2) Given that $p \neq q$ write down the number of solutions
of the simultaneous equations:

$$
\begin{aligned}
& y=p x+1 \\
& y=q x+4
\end{aligned}
$$

(3) Fred says that the simultaneous equations:

$$
\begin{aligned}
& y=m x \\
& y=a x^{2}+b x+c
\end{aligned}
$$

have 1 set of solutions. Is Fred right or could he be wrong?
(4) The lines $x=a$ and $y=b$ meet at the point $P$.
(a) Write down the coordinates of the point $P$.
(b) Find the distance of the point $P$ from the origin.
(5) The diagram below shows the graphs of two linear functions and two quadratic functions.
The equations of the quadratic functions are:

(a) Use the diagram to estimate the solutions of the
simultaneous equations:

$$
\begin{aligned}
& y=-1+4 x-x^{2} \\
& y=2\left(x^{2}-2\right)
\end{aligned}
$$

(b) Use the diagram to find the integer solutions of the equation: $3-x=2\left(x^{2}-2\right)$
(c) Using the diagram, write down the number of solutions to the equation $2\left(x^{2}-2\right)=2 x+1$
The two linear functions are $\mathrm{f}(x)$ and $\mathrm{g}(x)$.
(d) Use the graph to estimate the solutions of the simultaneous equations:

$$
\begin{aligned}
& y=\mathrm{f}(x) \\
& y=\mathrm{g}(x)
\end{aligned}
$$

(e) Use algebra to find the percentage error between your estimates found in part (d) and the actual solutions of the simultaneous equations.
(6) The diagram below shows the graphs of two equations.


The points $A(p, q)$ and $B(r, s)$ are two of the points where the graphs intersect.
(a) Using the graphs above, find the solutions to the
simultaneous equations:

$$
\begin{aligned}
& y=\frac{1}{x^{2}}+1 \\
& x^{2}+y^{2}=25
\end{aligned}
$$

giving your answers in terms of $p, q, r$ and $s$.
(b) Write down the number of solutions to the
simultaneous equations:

$$
\begin{aligned}
& x^{2}+y^{2}=25 \\
& y=x
\end{aligned}
$$

There are 2 real solutions to the simultaneous
equations:

$$
\begin{aligned}
& x^{2}+y^{2}=25 \\
& x=a
\end{aligned}
$$

(c) Complete the inequality $\qquad$ $<a<$ $\qquad$ .
(7) The diagram below shows the graphs of a linear function and a quadratic function.


State the number of real solutions to each equation:

| $\mathrm{f}(x)=\mathrm{g}(x)$ | $-\mathrm{f}(x)=\mathrm{g}(x)$ | $\mathrm{f}(x)=\mathrm{g}(x)-2$ |
| :---: | :---: | :---: |
| $\mathrm{f}(x)=2 \mathrm{~g}(x)$ | $\mathrm{f}(x+10)=\mathrm{g}(x)$ | $\mathrm{f}(2 x)=\mathrm{g}(x)$ |

