CP2 Series

Answers
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① $p^2r^2+2pr+1-(p^2r^2-2pr+1)\equiv 8r$

ⓐ $4pr\equiv 8r$

$\therefore p=2$

ⓑ $r=1: \cancel{9}-1$

$r=2: \cancel{25}-\cancel{9}$

$r=3: 49-\cancel{25}$

$r=n-1: \cancel{(2n-1)^2}-\cancel{(2n-3)^2}$

$r=n: (2n+1)^2-\cancel{(2n-1)}$

$\therefore \sum 8r=(2n+1)^2-1$

$8\sum r\equiv 4n^2+4n+1-1$

$8\sum r=4n(n+1)$

$\sum r=\frac{4}{8}n(n+1)$

$\sum r=\frac{1}{2}n(n+1)$ (r.t.s)

ⓒ $\sum_{r=1}^{20}-\sum_{r=1}^{10}=10(21)-5(11)=155 \quad \therefore 8\times 155=\underline{1240}$

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② ⓐ $f(x)=\tan 2x$

$f'(x)=2\sec^2(2x)$

$f''(x)=4\sec(2x)\times 2\sec(2x)\tan 2x$

$=8\sec^2(2x)\tan 2x$

u	du
$\sec^2(2x)$	$2(\sec 2x)\times 2\sec 2x\tan 2x$
$\tan 2x$	$2\sec^2(2x)$
v	dv

$f'''(x)=8\left[2\sec^2 2x\tan^2 2x+2\sec^4(2x)\right]$

$=16\sec^2 2x\tan^2 2x+16\sec^4(2x)$

ⓑ $f(0)=0$

$f'(0)=2$

$f''(0)=0$

$f'''(0)=16$

$\therefore f(0)+f'(0)(x)+\frac{f''(0)(x^2)}{2!}+\frac{f'''(0)(x^3)}{3!}$

$=0+2x+0+\frac{16x^3}{6}$

$=2x+\frac{8}{3}x^3$

③ ⓐ $\ln\left(\frac{1+2x}{\sqrt{1+x}}\right) \equiv \ln(1+2x) - \frac{1}{2}\ln(1+x)$

$\ln(1+2x) \equiv 2x - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} + \ldots$

$\frac{1}{2}\ln(1+x) \equiv \frac{1}{2}\left[x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots\right]$

$\therefore \ln(1+2x) - \frac{1}{2}\ln(1+x) = \left(2x - 2x^2 + \frac{8}{3}x^3\right) - \left(\frac{1}{2}x - \frac{1}{4}x^2 + \frac{1}{6}x^3\right)$

$= \frac{3}{2}x - \frac{7}{4}x^2 + \frac{5}{2}x^3, \quad \frac{1}{2} < |x| \leq \frac{1}{2}$

ⓑ $\frac{1}{2} < |x| \leq \frac{1}{2}$ ∴ his chace of 0.25 is fine ☺

ⓒ $\ln\left(\frac{0.5+x}{\sqrt{1+x}}\right) \equiv \ln\frac{1}{2}\left(\frac{1+2x}{\sqrt{1+x}}\right) = \ln\left(\frac{1}{2}\right) + \ln\left(\frac{1+2x}{\sqrt{1+x}}\right) \therefore \ln\left(\frac{1}{2}\right) + K$

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④ ⓐ $\frac{2}{r^2-1} \equiv \frac{A}{r+1} + \frac{B}{r-1}$ $\quad \therefore \frac{1}{r-1} - \frac{1}{r+1}$

$2 \equiv A(r-1) + B(r+1)$

let $r=1$: $2 = 2B$, $1 = B$

let $r=-1$: $2 = -2A$, $A = -1$

ⓑ
$r=2: \ 1 - \cancel{\frac{1}{3}}$

$r=3: \ \frac{1}{2} - \cancel{\frac{1}{4}}$

$r=4: \ \cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}$

$r=n-1: \ \cancel{\frac{1}{n-2}} - \frac{1}{n}$

$r=n: \ \cancel{\frac{1}{n-1}} - \frac{1}{n+1}$

$\therefore \sum_{r=2}^{20} \frac{2}{r^2-1} = 1 + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}$

$= \frac{3}{2} - \left[\frac{2n+1}{n(n+1)}\right]$

$= \frac{3}{2} - \frac{41}{20\times 21}$

$= \frac{589}{420}$ ✓

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⑤ $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$

$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120}$

$e^{\sin x} = e^{x - \frac{x^3}{6} + \ldots} = e^x \times e^{-\frac{x^3}{6}}$

$= \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right)\left(1 + \left(-\frac{x^3}{6}\right) + \ldots\right)$

$= 1 - \cancel{\frac{x^3}{6}} + x + \frac{x^2}{2} + \cancel{\frac{x^3}{6}} + \ldots$

$= 1 + x + \frac{x^2}{2} + 0x^3 \ldots$

6 a $\frac{1}{r!} - \frac{1}{(r+1)!} = \frac{(r+1)! - r!}{r!(r+1)!} \equiv \frac{(r+1)r! - r!}{r!(r+1)!} = \frac{r!\boxed{(r+1)-1}}{r!(r+1)!} = \frac{\cancel{r!}\,r}{\cancel{r!}(r+1)!}$

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b $r=1 : \frac{1}{1} - \cancel{\frac{1}{2}}$

$r=2 : \cancel{\frac{1}{2}} - \cancel{\frac{1}{6}}$

$r=3 : \cancel{\frac{1}{6}} - \cancel{\frac{1}{24}}$

....

$r=(n-1) : \cancel{\frac{1}{(n-1)!}} - \cancel{\frac{1}{n!}}$

$r=(n) : \cancel{\frac{1}{n!}} - \frac{1}{(n+1)!}$

$\therefore \sum_{r=1}^{n} \frac{r}{(r+1)!} = 1 - \frac{1}{(n+1)!} = \frac{(n+1)! - 1}{(n+1)!}$

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c $\sum_{r=4}^{5} = \sum_{r=1}^{5} - \sum_{r=1}^{3} = \frac{6! - 1}{6!} - \frac{4! - 1}{4!} = \frac{29}{720}$

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7 let $f(x) = \ln(1+e^x)$

$f'(x) = \frac{1}{1+e^x} \times e^x = \frac{e^x}{1+e^x}$

$f''(x) = \frac{e^x(1+e^x) - e^{2x}}{(1+e^x)^2} \equiv \frac{e^x}{(1+e^x)^2}$

u	$\frac{du}{dx}$
e^x	e^x
$1+e^x$	e^x
v	$\frac{dv}{dx}$

$f(0) = \ln(1+1) = \ln(2)$

$f'(0) = \frac{e^0}{1+e^0} = \frac{1}{1+1} = \frac{1}{2}$

$f''(0) = \frac{e^0}{(1+e^0)^2} = \frac{1}{(1+1)^2} = \frac{1}{2^2} = \frac{1}{4}$

Maclaurin in FB!

$f(0) + f'(0)x + f''(0)\frac{x^2}{2!}$

$\therefore \ln(1+e^x) \simeq \ln(2) + \frac{1}{2}(x) + \frac{1}{4}\left(\frac{x^2}{2}\right)$

$= \ln(2) + \frac{1}{2}x + \frac{1}{8}x^2$