Core Pure 1 Series Test – www.m4ths.com – Steve Blades ©

Answers

(1) Find the value of p given that:

$$\sum_{r=1}^{p-1} r = 78$$

(2) Show that

$$\sum_{r=1}^{k+1} r(3r-1) = (k+1)^2(k+2)$$

(3) Evaluate

$$\sum_{r=19}^{36} 4r - 3r^3$$

(4) $f(r) = pr + q$ where p and q are constants. Show that the sum given below is a multiple of 9

$$\sum_{r=5}^{40} f(r)$$

(5) Evaluate

$$\sum_{r=1}^{48} \left(\frac{r^3 + r^2 + r + 1}{2}\right)$$

(6) Find the value of k such that

$$\sum_{r=1}^{k} 3r^2 - \sum_{r=1}^{k} 17r = 0$$

(7) Find the least value of p such that

$$\sum_{r=1}^{p} r^3 - r^2 > 0$$

(8) $f(r) = 10r^2 - 29r - 3$

Given that $f(k) = 3417$, show that

$$\sum_{r=1}^{k} f(r) = 22550$$

Series Test

① $\sum r = \frac{1}{2}n(n+1)$

$$\therefore \sum_{r=1}^{p-1} = \frac{1}{2}(p-1)(p)$$

$$\frac{1}{2}(p-1)(p) = 78$$

$$(p-1)(p) = 156$$

$$p^2 - p - 156 = 0$$

$$(p-13)(p+12) = 0$$

$$\underline{p = 13} \quad p \neq -12$$

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②

$$3\sum_{1}^{K+1} r^2 - \sum_{1}^{K+1} r$$

$$3\left[\frac{1}{6}(K+1)(K+2)(2K+3)\right] - \left[\frac{1}{2}(K+1)(K+2)\right]$$

$$\frac{1}{2}(K+1)(K+2)(2K+3) - \frac{1}{2}(K+1)(K+2)$$

$$\frac{1}{2}(K+1)(K+2)\left[2K+3-1\right]$$

$$\frac{1}{\cancel{2}}(K+1)(K+2) \times \cancel{2}(K+1)$$

$$(K+1)(K+2)(K+1)$$

$$(K+1)^2(K+2)$$

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③ $4\sum r - 3\sum r^3 = 4\left[\frac{1}{2}(n)(n+1)\right] - 3\left[\frac{1}{4}(n)^2(n+1)^2\right]$

$$\sum_{1}^{36} - \sum_{1}^{18} = \left[2(36)(37) - \frac{3}{4}(36)^2(37)^2\right] - \left[2(18)(19) - \frac{3}{4}(18)^2(19)^2\right]$$

$$= -1240965$$

(4) $p\sum_{r=5}^{40} r + q\sum_{r=5}^{40} 1$ $\qquad \sum r = \frac{1}{2}n(n+1) \qquad \sum 1 = n$

$$\sum_{r=1}^{40} - \sum_{r=1}^{4}$$

$$p\left[\frac{1}{2}(40)(41)\right] + q[40] - \left[p\left[\frac{1}{2}(4)(5)\right] + q[(4)]\right]$$

$$20p(41) + 40q - 10p - 4q$$

$$10p(82-1) + 4q(10-1)$$

$$10p(81) + 9(4q)$$

$$9[90p + 4q] \checkmark$$

(5) $\frac{1}{2}\left[\frac{1}{4}(n)^2(n+1)^2 + \frac{1}{6}(n)(n+1)(2n+1) + \frac{1}{2}(n)(n+1) + n\right]$

$$\frac{1}{2}\left[\frac{1}{4}(48)^2(49)^2 + \frac{1}{6}(48)(49)(97) + \frac{1}{2}(48)(49) + 48\right] = 711112$$

(6) $3\sum r^2 - 17\sum r$

$$3\left[\frac{1}{6}n(n+1)(2n+1)\right] - 17\left[\frac{1}{2}n(n+1)\right]$$

$$\frac{1}{2}K(K+1)(2K+1) - \frac{17}{2}(K)(K+1) = 0$$

$$\frac{1}{2}K(K+1)[2K+1-17] = 0$$

$$\frac{1}{2}K(K+1)(2K-16) = 0$$

$$K(K+1)(K-8) = 0$$

$$K \neq 0 \quad K \neq -1 \quad \therefore \underline{K=8}$$

⑦ $\sum r^3 = \frac{1}{4}(n)^2(n+1)^2$

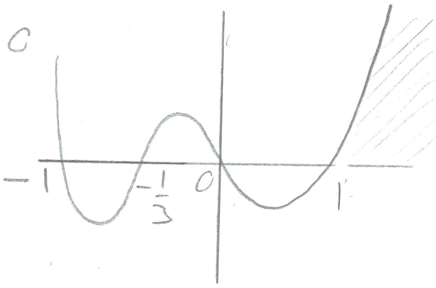
$\sum r^2 = \frac{1}{6}(n)(n+1)(2n+1)$

$\therefore \frac{1}{4}(p)^2(p+1)^2 - \frac{1}{6}(p)(p+1)(2p+1) > 0$

$\frac{1}{12}(p)(p+1)\left[3p(p+1) - 2(2p+1)\right] > 0$

$\frac{1}{12}(p)(p+1)\left[3p^2 - p - 2\right] > 0$

$\frac{1}{12}(p)(p+1)(3p+2)(p-1) > 0$



$\underline{\underline{p = 2}}$

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⑧ $10K^2 - 29K - 3 = 3417$

$10K^2 - 29K - 3420$

$(10K + 171)(K - 20) = 0$

$K \neq -\frac{171}{10} \quad \therefore K = 20$

$$\sum_{r=1}^{20} 10r^2 - 29r - 3 \quad = 10\sum_{r=1}^{20} r^2 - 29\sum_{r=1}^{20} r - 3\sum_{r=1}^{20} 1$$

$$= 10\left[\frac{1}{6}(n)(n+1)(2n+1)\right] - 29\left[\frac{1}{2}(n)(n+1)\right] - 3\left[n\right]$$

$$10\left(\frac{1}{6}(20)(21)(41)\right) - 29\left(\frac{1}{2}(20)(21)\right) - 3(20)$$

$$28700 \quad - 6090 \quad - 60$$

$\underline{\underline{22550}}$ ✓