A Level Furter Maths - CP2 Chapter 2 - Series – www.m4ths.com

(1) (a) Given that $(pr+1)^2 - (pr-1)^2 \equiv 8r$ where $p \in Z^+$, find the value of p.

(b) Using the method of differences, show that:

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

(c) Hence, evaluate:

$$\sum_{r=11}^{20} (pr+1)^2 - (pr-1)^2$$

(2) $f(x) = \tan 2x$

(a) Find an expression for $f'(x), f''(x)$ and $f'''(x)$.

(b) Hence, find the first 2 non-zero terms in the Maclaurin Series expansion of $\tan 2x$ giving each term in its simplest form.

(3) (a) Use the series expansion of $\ln(1+x)$ to find the first 3 terms in the expansion of $\ln\frac{1+2x}{\sqrt{1+x}}$.

(b) Fred wants to use the expansion to find an estimate for $\ln\frac{1.5}{\sqrt{1.25}}$. Comment on his approach.

(c) Given that when using a Maclaurin expansion, a numerical approximation of $\ln\frac{1+2x}{\sqrt{1+x}}$ is k, find an exact expression for an approximation of $\ln\frac{0.5+x}{\sqrt{1+x}}$, explaining your reasoning.

(4) (a) Express $\frac{2}{r^2-1}$ in partial fractions.

(b) Use the method of differences to show that:

$$\sum_{r=2}^{20} \frac{2}{r^2-1} = \frac{589}{420}$$

(5) Using the Maclaurin expansions for e^x and $\sin x$, show that the expansion of $e^{\sin x}$ is independent of the term in x^3

(6) (a) Show that $\frac{1}{r!} - \frac{1}{(r+1)!}$ can be written as $\frac{r}{(r+1)!}$

(b) Use the method of differences to show that:

$$\sum_{r=1}^{n} \frac{r}{(r+1)!} = \frac{(n+1)!-1}{(n+1)!}$$

(c) Hence, show that:

$$\sum_{r=4}^{5} \frac{r}{(r+1)!} = \frac{29}{720}$$

(7) Use differentiation to show that the first 3 terms in the Maclaurin series expansion of $\ln(1+e^x)$ are $\ln 2 + \frac{x}{2} + \frac{x^2}{8}$