Hyperbolics Answers

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① $2(\cosh^2 x - 1) - 5\cosh x - 1 = 0$

$2\cosh^2 x - 5\cosh x - 3 = 0$

$(2c+1)(c-3) = 0$

$\cosh x \neq -\frac{1}{2}$ $\therefore$ $\underline{\cosh x = 3}$

$\downarrow$

$x = \ln(3+\sqrt{8})$

$x = \ln(3-\sqrt{8})$ $\underline{\underline{0.e}}$

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② ⓐ $f(x) = \sinh(2x)$ $\quad f(0) = 0$

$f'(x) = 2\cosh(2x)$ $\quad f'(0) = 2$

$f''(x) = 4\sinh(2x)$ $\quad f''(0) = 0$

$f'''(x) = 8\cosh(2x)$ $\quad f'''(0) = 8$

$f^{iv}(x) = 16\sinh(2x)$ $\quad f^{iv}(0) = 0$

$f^{v}(x) = 32\cosh(2x)$ $\quad f^{v}(0) = 32$

$f(x) = f(0) + f'(0)x + f'' \ldots$

$\therefore \sinh(2x) \approx 0 + 2x + 0x^2 + \frac{8}{6}x^3 + 0x^4 + \frac{32}{120}x^5$

$= 2x + \frac{4}{3}x^3 + \frac{4}{15}x^5 + \ldots$

ⓑ let $x = 0.4$ $\therefore$ $\sinh(0.8) \approx 2(0.4) + \frac{4}{3}(0.4)^3 + \frac{4}{15}(0.4)^5$

$\approx \underline{0.888064}$

$\therefore 0.00473\%$

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③ $\text{artanh}\, x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

$\therefore 2\left[\frac{1}{2}\ln\left(\frac{1+p}{1-p}\right)\right] = \ln q$

$\frac{1+p}{1-p} = q$

$1+p = q(1-p)$

$p + pq = q - 1$

$p(1+q) = q - 1$

$p = \frac{q-1}{q+1}$ o.e.

④ $\cosh^2 A = 1 + \sinh^2 A$

$\cosh^2 A = 1 + \frac{4}{25}$

$\cosh^2 A = \frac{29}{25}$

$\cosh A = \pm\frac{\sqrt{29}}{5}$

$\cosh A = \frac{\sqrt{29}}{5}$

$\cosh A > 0$

$\sinh 2A = 2\sinh A\cosh A$

$\therefore 2\left(\frac{2}{5}\right)\left(\frac{\sqrt{29}}{5}\right) = \frac{4\sqrt{29}}{25}$

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⑤ $\int e^x\left(\frac{e^x + e^{-x}}{2}\right)dx$

$\frac{1}{2}\int\left(e^{2x} + 1\right)dx$

$\frac{1}{2}\left[\frac{1}{2}e^{2x} + x\right]_{\ln 4}^{\ln 8}$

$\frac{1}{2}\left[\left(\frac{1}{2}e^{\ln 64} + \ln 8\right) - \left(\frac{1}{2}e^{\ln 16} + \ln 4\right)\right]$

$\frac{1}{2}\left[\left(\frac{1}{2}(64) + 3\ln 2\right) - \left(\frac{1}{2}(16) + 2\ln 2\right)\right]$

$\frac{1}{2}\left[(32 + 3\ln 2) - (8 + 2\ln 2)\right]$

$\frac{1}{2}\left[24 + \ln 2\right]$

$12 + \frac{1}{2}\ln 2$

$12 + \ln 2^{\frac{1}{2}}$

$12 + \ln\sqrt{2}$

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⑥ $y = \cosh x - 3\cosh 2x$

$\frac{dy}{dx} = \sinh x - 6\sinh 2x$

For SP: $\frac{dy}{dx} = 0$

$0 = \sinh x - 6\sinh 2x$

$0 = \sinh x - 12\sinh x\cosh x$

$0 = \sinh x(1 - 12\cosh x)$

$\downarrow \qquad \downarrow$

$\sinh x = 0 \qquad \cosh x \neq \frac{1}{12}$

$\therefore x = 0$

When $x = 0$, $y = \cosh(0) - 3\cosh(0)$
$= -2$

$\therefore SP(0, -2)$

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(7) $\cosh(2A) \equiv \frac{e^{2A} + e^{-2A}}{2}$ and $\cosh(A) = \frac{e^{A} + e^{-A}}{2}$

RHS $2\left(\frac{e^{A} + e^{-A}}{2}\right)^2 - 1 \equiv 2\left(\frac{e^{A} + e^{-A}}{2}\right)\left(\frac{e^{A} + e^{-A}}{2}\right) - 1$

$= \cancel{2}\left(\frac{e^{2A} + 1 + 1 + e^{-2A}}{\cancel{(2)}(2)}\right) - 1$

$\equiv \frac{e^{2A} + e^{-2A} + 2}{2} - 1$

$\equiv \frac{e^{2A} + e^{-2A}}{2} + 1 - 1$

$\equiv \frac{e^{2A} + e^{-2A}}{2}$ ✓ RHS = LHS ✓

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(8) $\frac{e^{x} - e^{-x}}{2} + \frac{e^{x} + e^{-x}}{2} = 7$

$e^{x} = 7$

$x = \ln 7$

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(9) $y = \text{artanh}(x)$

$\tanh y = x$

$\frac{e^{2y} - 1}{e^{2y} + 1} = x$

$$e^{2y}-1 = xe^{2y}+x$$

$$e^{2y}-xe^{2y} = 1+x$$

$$e^{2y}(1-x) = 1+x$$

$$e^{2y} = \frac{1+x}{1-x}$$

$$2y = \ln\left|\frac{1+x}{1-x}\right|$$

$$y = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|$$

$$\therefore \operatorname{artanh}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad |x|<1.$$

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(10) $$\int \frac{1}{\sqrt{(x-1)^2+9}}\,dx$$

let $x-1 = 3\sinh u$.

$$\frac{dx}{du} = 3\cosh u$$

$$dx = 3\cosh u\,du$$

$$x-1 = 3\sinh u$$

$$\frac{x-1}{3} = \sinh u$$

$$\operatorname{arsinh}\left(\frac{x-1}{3}\right) = u$$

$$\int \frac{1}{\sqrt{(3\sinh u)^2+9}} \times 3\cosh u\,du$$

$$= \int \frac{1}{3\cosh u} \times 3\cosh u\,du$$

$$= \int 1\,du$$

$$= u + C$$

$$= \operatorname{arsinh}\left(\frac{x-1}{3}\right) + c$$

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(11) $$\int \sinh x(\sinh^2 x)\,dx$$

$$= \int \sinh x(\cosh^2 x - 1)\,dx$$

$$= \int \sinh x\cosh^2 x - \sinh x\,dx$$

$$= \frac{1}{3}\cosh^3 x - \cosh x + c$$

$$= \frac{1}{3}\cosh x\left[\cosh^2 x - 3\right] + c \quad \checkmark$$

(12) $y = \operatorname{arcosh} x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x^2-1}} = (x^2-1)^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2}(x^2-1)^{-\frac{3}{2}} \times 2x$$

$$= \frac{-x}{(x^2-1)^{3/2}}$$

$$(x^2-1)\left(\frac{-x}{(x^2-1)^{3/2}}\right) + x\left(\frac{1}{(x^2-1)^{1/2}}\right)$$

$$\frac{-x}{(x^2-1)^{\frac{1}{2}}} + \frac{x}{(x^2-1)^{\frac{1}{2}}} = 0 \checkmark$$

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(13) $$\int_0^4 \frac{2x}{\sqrt{x^2+16}} + \frac{1}{\sqrt{x^2+16}}\,dx$$

$$= \int_0^4 2x(x^2+16)^{-\frac{1}{2}}\,dx + \left[\operatorname{arsinh}\left(\frac{x}{4}\right)\right]_0^4$$

$$= \left[2\sqrt{x^2+16} + \operatorname{arsinh}\left(\frac{x}{4}\right)\right]_0^4$$

$$= \left(2\sqrt{32} + \operatorname{arsinh}(1)\right) - \left(2(4) + \operatorname{arsinh}(0)\right)$$

$$= 8\sqrt{2} + \ln(1+\sqrt{2}) - 8 + 0$$

$$= 8(\sqrt{2}-1) + \ln(1+\sqrt{2}) \checkmark$$

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(14) (a) $R\cosh x \cosh\alpha + R\sinh x \sinh\alpha \equiv 10\cosh x + 6\sinh x$

$\therefore R\cosh\alpha = 10$ and $R\sinh\alpha = 6 \Rightarrow R^2 = 10^2 - 6^2 \Rightarrow R = 8$

$\tanh\alpha = \frac{6}{10} \Rightarrow \alpha = 0.693.$ $8\cosh(x + 0.693)$

(b) $\frac{1}{8}$