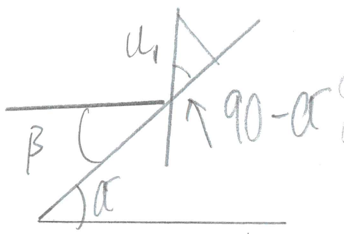
www.m4ths.com Oblique Impacts

① 

$$\begin{array}{l} S\ 1 \\ \downarrow\ u\ 0 \\ v\ u_1 \\ A\ g \\ T \end{array} \qquad \begin{array}{l} u_1^2 = u^2 + 2as \\ u_1^2 = 0 + 2g(1) \\ u_1^2 = 2g \end{array}$$

I have set 'v' as u_1 on the initial impact.

$$\tan\alpha = 2 \qquad \sin\alpha = \frac{2}{\sqrt{5}} \qquad \cos\alpha = \frac{1}{\sqrt{5}}$$

$$\left[\begin{array}{l} v\cos(\beta) = u\cos(90-\alpha) \\ v\sin(\beta) = eu\sin(90-\alpha) \end{array}\right.$$

$$\left[\begin{array}{l} v\cos(\beta) = \sqrt{2g}\sin(\alpha) \\ v\sin(\beta) = \frac{1}{\sqrt{2}} \times \sqrt{2g}\cos(\alpha) \end{array}\right.$$

$$\left[\begin{array}{l} v\cos(\beta) = \frac{2\sqrt{2g}}{\sqrt{5}} \\ v\sin(\beta) = \frac{1}{\sqrt{2}}\sqrt{2g} \times \frac{1}{\sqrt{5}} \end{array}\right.$$

$$\left[\begin{array}{l} v\cos(\beta) = \frac{2\sqrt{2g}}{\sqrt{5}} \\ v\sin(\beta) = \frac{\sqrt{g}}{\sqrt{5}} \end{array}\right.$$

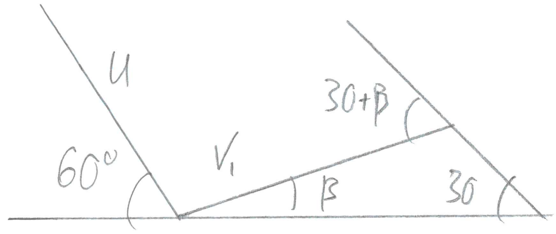
$$v^2\cos^2\beta + v^2\sin^2\beta = \left(\frac{2\sqrt{2g}}{\sqrt{5}}\right)^2 + \left(\frac{\sqrt{g}}{\sqrt{5}}\right)^2$$

$$v^2(\cos^2\beta + \sin^2\beta) = \frac{8g}{5} + \frac{g}{5}$$

$$v^2(1) = \frac{9g}{5}$$

$$v^2 = \frac{9}{5}g \checkmark$$

(2)



Bounce 1

$$\begin{cases} V_1 \cos\beta = u\cos 60 \\ V_1 \sin\beta = eu\sin 60 \end{cases}$$

$$\begin{cases} V_1 \cos\beta = 6\left(\frac{1}{2}\right) \\ V_1 \sin\beta = \frac{4}{5}(6)\left(\frac{\sqrt{3}}{2}\right) \end{cases}$$

$$\begin{cases} V_1 \cos\beta = 3 \\ V_1 \sin\beta = \frac{12}{5}\sqrt{3} \end{cases}$$

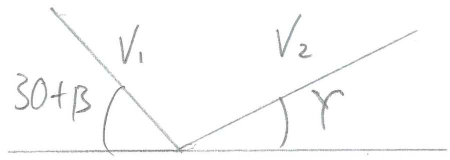
$$V_1 = \sqrt{3^2 + \left(\frac{12}{5}\sqrt{3}\right)^2}$$

$$V_1 = \frac{3\sqrt{73}}{5} \quad (A)$$

$$\beta = \tan^{-1}\left(\frac{\frac{12\sqrt{3}}{5}}{3}\right)$$

$$\beta = 54.1\ldots \quad (B)$$

Bounce 2



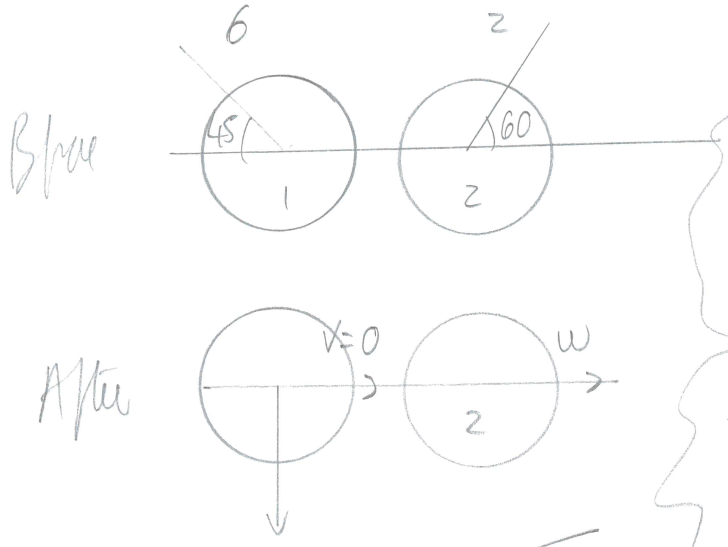
$$\begin{cases} V_2 \cos\gamma = V_1 \cos(30+\beta) \\ V_2 \sin\gamma = \frac{4}{5} V_1 \sin(30+\beta) \end{cases}$$

$$\begin{cases} V_2 \cos\gamma = \frac{3\sqrt{73}}{5}\cos(84\ldots) \\ V_2 \sin\gamma = \frac{12\sqrt{73}}{25}\sin(84\ldots) \end{cases}$$

$$V_2^2 = \left(\left(\frac{3\sqrt{73}}{5}\cos(84..)\right)^2 + \left(\frac{12\sqrt{73}}{25}\sin 84\right)^2\right)$$

$$V_2 = 4.11 \text{ ms}^{-1}$$

(3)



→ C.O.L.M

$1(6\cos45) - 2(2\cos60) = 0 + 2w$

$3\sqrt{2} - 2 = 2w$ (1)

e $\quad \frac{w-0}{6\cos45 + 2\cos60} = e$

$\frac{w}{3\sqrt{2}+1} = e$

$w = e(3\sqrt{2}+1)$ (2)

Simultaneous Equations

$3\sqrt{2} - 2 = 2w$ (1)

$3\sqrt{2} - 2 = 2e(3\sqrt{2}+1)$

$\frac{3\sqrt{2}-2}{2(3\sqrt{2}+1)} = e$

$\frac{20-9\sqrt{2}}{34} = e$

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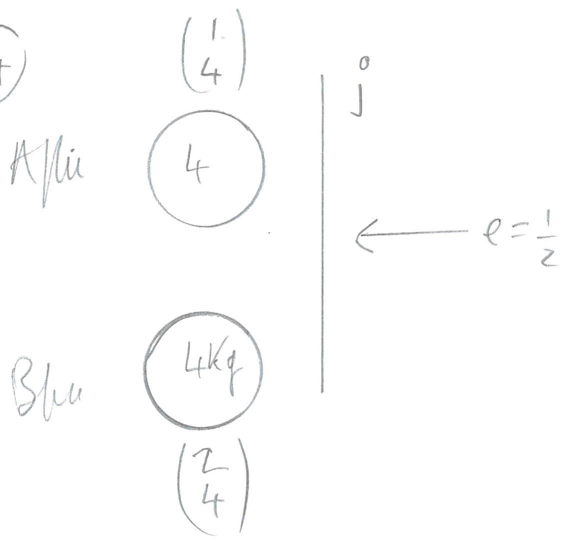
(4)

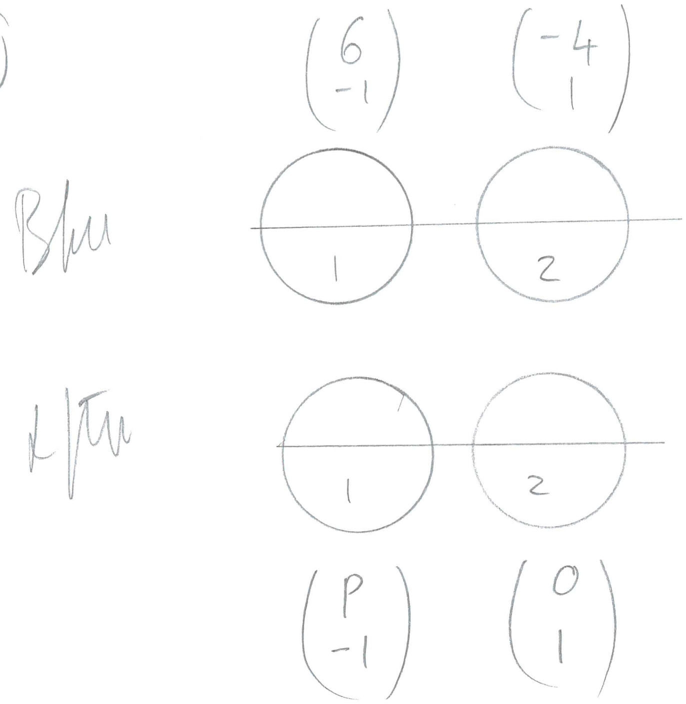


Only need to consider K.E loss ⊥ to j

K.E Before $\qquad$ K.E After

$\frac{1}{2} \times 4 \times (2^2) - \frac{1}{2} \times 4 (1)^2 = 6J$

(5)



C.O.L.M to find p

$6(1) - 4(2) = p(1) + 0$

$p = -2$

$e = \frac{0 - -2}{6 - -4} = \frac{1}{5}$

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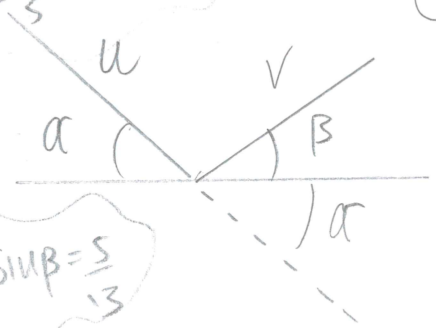
(6)

$\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$

$\tan\beta = \frac{5}{12}$

$\cos\beta = \frac{12}{13}$, $\sin\beta = \frac{5}{13}$



(a)

$v\cos\beta = u\cos\alpha$

$v\sin\beta = eu\sin\alpha$

$v\left(\frac{12}{13}\right) = u\left(\frac{4}{5}\right)$ ①

$v\left(\frac{5}{13}\right) = eu\left(\frac{3}{5}\right)$ ②

② ÷ ①

$\frac{3e}{4} = \frac{5}{12} \Rightarrow e = \frac{20}{36} = \frac{5}{9}$

(b)

$\theta = \alpha + \beta$

$\tan\theta = \tan(\alpha + \beta)$

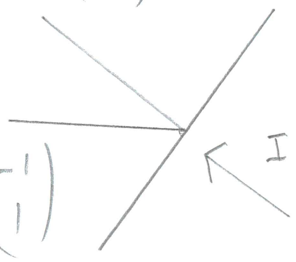
$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \left(\frac{3}{4}\right)\left(\frac{5}{12}\right)}$

$= \frac{56}{33}$ ✓

⑦ Bfore $\begin{pmatrix}3\\-2\end{pmatrix}$

Aftr. $\begin{pmatrix}-1\\1\end{pmatrix}$



$I = m(v-u)$

$I = 2\left(\begin{pmatrix}-1\\1\end{pmatrix} - \begin{pmatrix}3\\-2\end{pmatrix}\right)$

$I = 2\begin{pmatrix}-4\\3\end{pmatrix}$

NB you can use a unit vector for the next part!

$-e \cdot u \cdot I = v \cdot I$

$-e\begin{pmatrix}3\\-2\end{pmatrix}\cdot\begin{pmatrix}-4\\3\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix}\cdot\begin{pmatrix}-4\\3\end{pmatrix}$

$-e\left[-12-6\right] = \left[4+3\right]$

$-e(-18) = 7$

$e = \frac{7}{18}$

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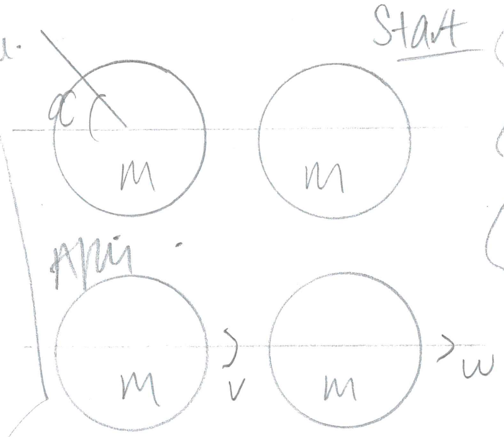
⑧ Bfore. Start

$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

Aftr.



C.O.L.M →

$mu\cos\alpha + 0 = mv + mw$

$u\cos\alpha = v + w$

$\frac{4}{5}u = v + w$ ①

e →

$\frac{w-v}{u\cos\alpha - 0} = \frac{1}{2}$

$\frac{w-v}{\frac{4}{5}u} = \frac{1}{2}$

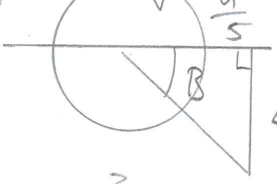
$w - v = \frac{2}{5}u$ ②

① - ②

$2v = \frac{2}{5}u$

$v = \frac{u}{5}$

After



← $u\sin\alpha$ $\frac{3}{5}u$

$\tan\beta = \frac{\frac{3}{5}u}{\frac{u}{5}}$

$\tan\beta = 3$

Angle of Deflection

$\gamma = \beta - \alpha$

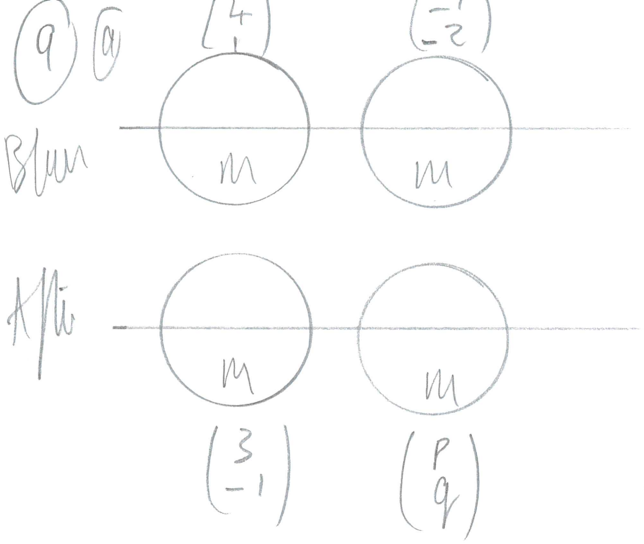
$\tan\gamma = \tan(\beta - \alpha)$

$\tan\gamma = \tan\beta - \tan\alpha / 1 + \tan\beta\tan\alpha$

$\tan\gamma = \frac{3 - \frac{3}{4}}{1 + 3\left(\frac{3}{4}\right)}$

$\tan\gamma = \frac{9}{13}$

9 (a)



C.O.LM i's

$4m - m = 3m + pm$

$4 - 1 = 3 + p$

$3 = 3 + p$

$0 = p$

C.O.LM j's

$1m + 2m = -m + qm$

$1 - 2 = -1 + q$

$-1 = -1 + q$

$0 = q$

OR $I = m(v-u)$ for A and apply opposite to B

(b) $I = m(v-u)$ for either.

B is easier as its brought to rest.

For A $\quad I = m(v-u)$

$I = m\left(\binom{3}{-1} - \binom{4}{1}\right)$

$I = m\binom{-1}{-2}$

you can see this here!

This is a multiple of the unit vector

'Speed' of A in direction of Impulse Before $\binom{4}{1} \cdot \binom{-1}{-2} = -6$

'Speed' of B in direction of Impulse Before

5

'Speed' of A after

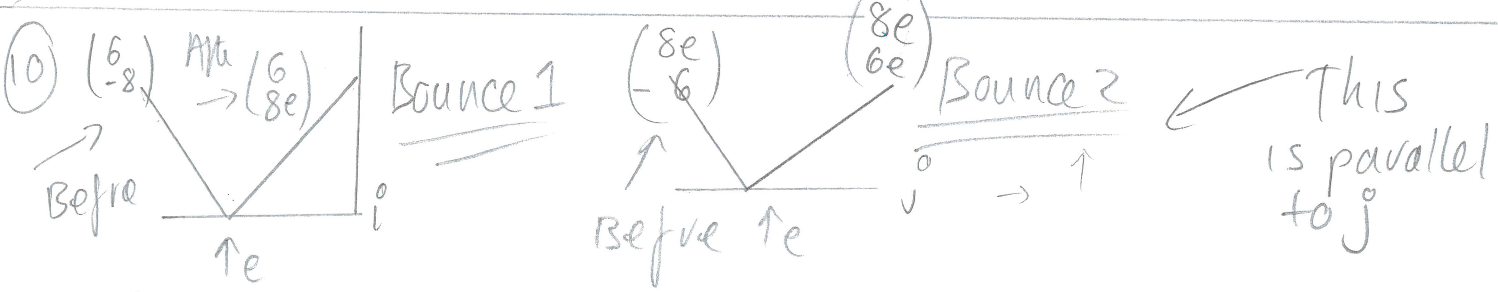
$\binom{3}{-1} \binom{-1}{-2} = -1$

Speed of B after

0

$\therefore \quad e = \dfrac{0 - -1}{5 - -6} = \dfrac{1}{11}$

10



After the second bounce the velocity vector is $\binom{-6e}{8e}$

$\therefore \binom{-6e}{8e} = \binom{K}{4}$ Solving for j $\quad 8e = 4$, $e = \frac{1}{2}$ $\quad$ solving for i $\quad -6(\frac{1}{2}) = K \Rightarrow K = -3$