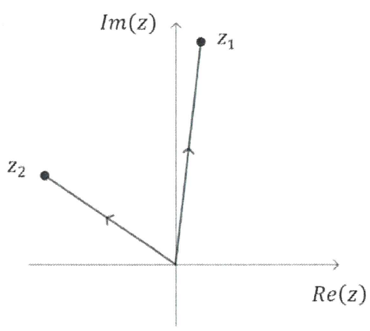
Core Pure 1 – Chapters 1-4 Test 3 – www.m4ths.com – Steve Blades ©

(1) The diagram shows the complex numbers z_1 and z_2 on an Argand diagram. Complete the table for the given statements ticking the relevant box.



Statement	True	False	More Information Needed
$\lvert z_1\rvert > \lvert z_2\rvert$			
$\arg(z_1) > \arg(z_2)$			
$\arg\left(\frac{z_2}{z_1}\right) < 0$			
$\lvert z_1 z_1\rvert < 3\lvert z_2\rvert$			
$\frac{-5\pi}{6} < \arg({z_2}^*) < \frac{-\pi}{2}$			
$\left\lvert\frac{z_1}{{z_1}^*}\right\rvert > \lvert z_2\rvert$			

(2) Show that the solution to the equation $2i(1+x) = x - 1$ can be written in the form $k(1+2i)^n$.

(3) The complex number z satisfies $\arg(z - 4 - 3i) = \frac{\pi}{4}$ and $|z| = \sqrt{113}$. Find z in the form $a + bi$.

(4) A quadratic equation has root α and β. Given that $\sum \alpha = -8$ and $\alpha\beta = 25$, find a quadratic equation with integer coefficients that has the roots $(2\alpha - 3)$ and $(2\beta - 3)$.

(5) Show that there are no values of k that satisfies the equation

$$\sum_{r=k+1}^{2k} 3r^2 = \sum_{r=2k+1}^{4k} r$$

(6) A quadratic equation has roots α and β. Given that $\alpha + \beta = -6$ and $\alpha^3 + \beta^3 = 18$, find the quadratic equation in the form $ax^2 + bx + c = 0$

(7) (a) On an Argand diagram, shade the region of points that satisfies

$$\{z \epsilon C: |z - 2i| < 4\} \cap \{z \epsilon C: |z + 6 - 6i| \leq |z - 6|\}$$

(b) Given that $i < Im(z) < 2i$ find two possible values p and q such that $p < \arg(z) < q$.

(8) $z_1 = \sqrt{3}\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$

(a) Write z_1 in the form $x + iy$

(b) A second complex number z_2 is such that $|z_1 z_2| = 6$ and $\arg(z_1 z_2) = 0$. Find z_2 in the form $x + iy$.

(9) $x^4 - 4x^3 + 11x^2 - 14x + 10 = 0$ has roots α, β, γ and δ. Find the value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$.

(10) Given that $z_1 = 6 + 3i$ and $z_1 - z_2 = 6i$, state the relation between z_1 and z_2.

① ① T
② F
③ F
④ Need more info
⑤ T
⑥ Need more info

② $2i + 2ix = x - 1$

$1 + 2i = x - 2ix$

$1 + 2i = x(1 - 2i)$

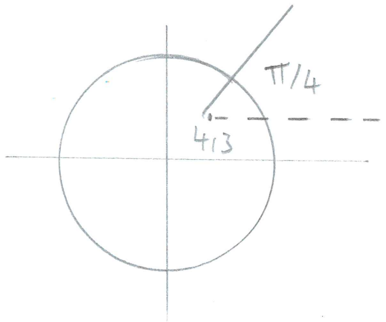
$\frac{1+2i}{1-2i} = x$

$\frac{(1+2i)(1+2i)}{(1-2i)(1+2i)} = x$

$\frac{(1+2i)^2}{1+4} = x$

$\frac{1}{5}(1+2i)^2 = x$

③



Circle: $x^2 + y^2 = 113$

Half Line: $y - 3 = 1(x - 4)$

$y = x - 1$

∴ $x^2 + (x-1)^2 = 113$

$x^2 + x^2 - 2x + 1 =$

$2x^2 - 2x + 1 = 113$

$x^2 - x - 56 = 0$

$(x-8)(x+7) = 0$

$x \neq -7$ ∴ $x = 8,\ y = 7$

∴ $z = 8 + 7i$

④ $x^2 + 8x + 25 = 0$ let $w = 2x - 3$

$\frac{w+3}{2} = x$

∴ $\left(\frac{w+3}{2}\right)^2 + 8\left(\frac{w+3}{2}\right) + 25 = 0$

$(w+3)^2 + 16(w+3) + 100 = 0$

$w^2 + 6w + 9 + 16w + 48 + 100 = 0$

$w^2 + 22w + 157 = 0$

⑤ $\sum_{r=K+1}^{2K} 3r^2 = 3\left[\frac{1}{6}(2K)(2K+1)(4K+1) - \frac{1}{6}(K)(K+1)(2K+1)\right]$

$= \frac{1}{2}K(2K+1)\left[2(4K+1) - (K+1)\right]$

$= \frac{1}{2}K(2K+1)(7K+1)$

$\sum_{r=2K+1}^{4K} = \frac{1}{2}(4K)(4K+1) - \frac{1}{2}(2K)(2K+1)$

$= \frac{1}{2}(2K)\left[2(4K+1) - (2K+1)\right]$

$= \frac{1}{2}(2K)\left[6K+1\right]$

∴ $\frac{1}{2}K(2K+1)(7K+1) = \frac{1}{2}(2K)(6K+1)$

$(2K+1)(7K+1) = 2(6K+1)$

$14K^2 + 9K + 1 = 12K + 2$

$14K^2 - 3K - 1 = 0$

K must be an integer.

$K = \frac{3 \pm \sqrt{65}}{26}$ ∴ no values of K.

⑥ $\alpha^3 + \beta^3 = (\Sigma\alpha)^3 - 3(\Sigma\alpha)(\alpha\beta)$

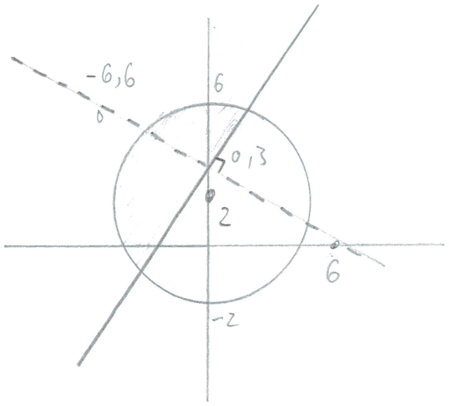
$18 = (-6)^3 - 3(-6)(\alpha\beta)$

$\alpha\beta = 13$

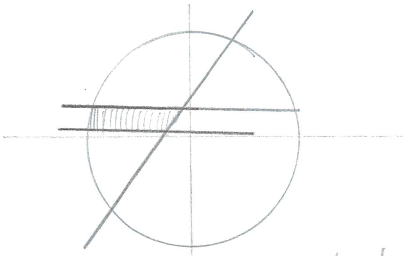
∴ $x^2 + 6x + 13 = 0$

(7)

(a)



(b)



$p=\frac{\pi}{2}$ $q=\pi$ are possibles.

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(8) (a) $\sqrt{3}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)=\frac{3}{2}-\frac{\sqrt{3}}{2}i$

(b) $|z_1||z_2|=6$ $\qquad$ $\arg(z_1z_2)=\arg z_1+\arg z_2$

$\sqrt{3}|z_2|=6$

$|z_2|=\frac{6}{\sqrt{3}}$ $\qquad$ $\therefore 0=-\frac{\pi}{6}+\arg z_2$

$\underline{|z_2|=2\sqrt{3}}$ $\qquad$ $\arg z_2=+\frac{\pi}{6}$

$\therefore 2\sqrt{3}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

$2\sqrt{3}\left(\frac{\sqrt{3}}{2}+i\left(\frac{1}{2}\right)\right)=\underline{3+\sqrt{3}i}$

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(9) $\alpha^2+\beta^2+\gamma^2+\delta^2\equiv(\Sigma\alpha)^2-2(\Sigma\alpha\beta)$

$\Sigma\alpha=4$ $\qquad$ $\therefore 16-2(11)=\underline{\underline{-6}}$

$\Sigma\alpha\beta=11$

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(10) $6+3i-z_2=6i$

$6-3i=z_2$

$\therefore z_2=z_1^*$

It's the complex conjugate.