Core Pure 1 – Chapters 1-4 Test 1 – www.m4ths.com – Steve Blades ©

(1) Write $4 - 4i$ in the form $r(\cos\theta + i\sin\theta)$

(2) the complex numbers z_1 and z_2 are such that $z_1 = -3 + 4i$ and $|z_1 z_2| = 10$. Given further that $\arg(z_2) = -\pi$,
(a) Express $z_1 - z_2$ in the form $a + bi$.
(b) Plot $z_1 - z_2$ on an Argand diagram.

(3) The complex numbers $a + bi$ and $c + di$ lie in the region which satisfies:

$$\{z\epsilon C: |z - 4 - 3i| \leq 4\} \cap \left\{z\epsilon C: \frac{\pi}{4} \leq \arg(z - 4 - 3i) \leq \frac{3\pi}{4}\right\}$$

Find the maximum value of $c - a$ in the form $p\sqrt{q}$.

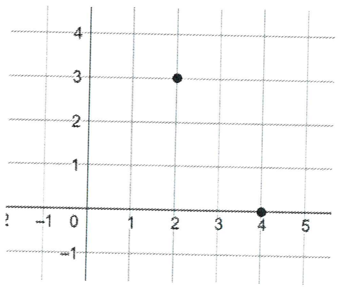
Answers

(4) A cubic equation has the roots α, β and γ.
Given that $\sum\alpha = 6, \sum\alpha\beta = 25$ and $\alpha\beta\gamma = 82$, find:
(a) $\alpha^2\beta^2\gamma^2$
(b) $\alpha^2 + \beta^2 + \gamma^2$
(c) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$
(d) An equation with integer coefficients that has the roots $\alpha - 3,\ \beta - 3$ and $\gamma - 3$.

(5) Find possible values of a, b, c, d and e for the sum given below

$$\sum_{r=2n+1}^{4n} r^2 = an(bn + c)(dn + e)$$

(6) A quartic equation has roots α, β, γ and δ. The two points shown on the Argand diagram below represent roots of the quartic equation.



Given that the quartic equation can be written in the form $(x - p)^2(x^2 + qx + r) = 0$, show that $\alpha\beta\gamma\delta = 208$.

(7) Show that

$$\sum_{r=1}^{n} 12r(r + 1)(r - 1) = 3n(n + 2)(n^2 - 1)$$

Answers FM Blades

(1) $|z| = 4^2+4^2$
$= 4\sqrt{2}$

$\arg(z) = -\frac{\pi}{4}$

$\therefore 4\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) - i\sin\left(\frac{\pi}{4}\right)\right)$

(2) $|z_1| = 5$

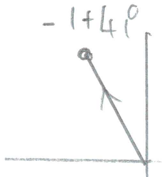
$|z_1||z_2| = |z_1 z_2| \therefore |z_2| = 2$

(c) $2(\cos(-\pi) + i\sin(-\pi)) = -2$

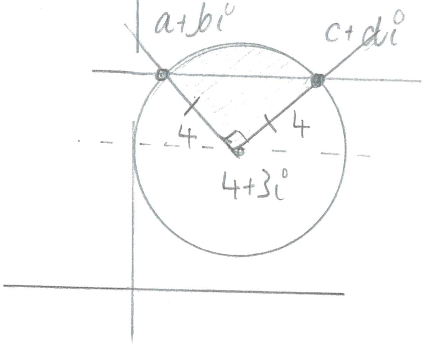
$z_2 = -2$

$z_1 - z_2 = -1+4i$

(b)



(3)



$c - a = \sqrt{4^2+4^2}$
$= 4\sqrt{2}$

(4) (a) $\alpha^2\beta^2\gamma^2 = (\alpha\beta\gamma)^2 = 82^2 = \underline{6724}$

(b) $\alpha^2+\beta^2+\gamma^2 = \left(\sum\alpha\right)^2 - 2\left(\sum\alpha\beta\right)$
$= (6)^2 - 2(25)$
$= 36 - 50$
$= \underline{-14}$

(c) $\frac{\alpha\beta+\alpha\gamma+\beta\gamma}{\alpha\beta\gamma} = \frac{25}{82}$

(d) $x^3 - 6x^2 + 25x - 82 = 0$

let $w = x - 3$
$w + 3 = x$

$\therefore (w+3)^3 - 6(w+3)^2 + 25(w+3) - 82 = 0$

$w^3 + 3(w^2)(3) + 3(w)(9) + 27$
$-6[w^2+6w+9] + 25w + 75 - 82 = 0$

$w^3 + 9w^2 + 27w + 27 - 6w^2 - 36w - 54$
$+ 25w + 75 - 82 = 0$

$w^3 + 3w^2 + 16w - 34 = 0$

(5) Formula Book

$\sum r^2 = \frac{1}{6}n(n+1)(2n+1)$

$\sum_1^{4n} r^2 = \frac{1}{6}(4n)(4n+1)(8n+1) - \sum_1^{2n} r^2 = \frac{1}{6}(2n)(2n+1)(4n+1)$

$= \frac{1}{6}(2n)(4n+1)[2(8n+1) - (2n+1)]$

$= \frac{1}{3}n(4n+1)[14n+1]$

$a = \frac{1}{3}$, $b = 4$, $c = 1$, $d = 14$, $e = 1$ (one possibility)

(6) $\alpha = 2+3i$
$\beta = 2-3i$
$\gamma = 4$
$\delta = 4$

$\alpha\beta\gamma\delta = (2+3i)(2-3i)(4)(4)$
$= (4 - 6i + 6i + 9)(16)$
$= (13)(16)$
$= 208$

(7) $\sum_1^n 12r(r^2-1) = 12\left(\sum r^3 - \sum r\right)$ Formula Book

$= 12\left[\frac{1}{4}n^2(n+1)^2 - \left(\frac{1}{2}\right)n(n+1)\right]$

$= 3n^2(n+1)^2 - 6n(n+1)]$

$= 3n(n+1)[n(n+1) - 2]$

$= 3n(n+1)[n^2+n-2]$

$= 3n(n+1)(n+2)(n-1)$

$= 3n(n^2-1)(n+2)$

$= 3n(n+2)(n^2-1)$ ✓