

(39) Methods of Proof

WORKING AT D/E

(1) Prove that the difference between any two prime numbers is not always an even number.

(2) If n is a single digit odd number, prove that $n + 1$ is not always a single digit even number.

(3) Prove, by counter example, that $2n^2 + 1$ for all positive integers n is not always a prime number.

WORKING AT B/C

(1) (a) Given that $2n$ is always even, show that the sum of the squares of two consecutive even numbers can be written as

$$8n^2 + 8n + 4$$

(b) Hence, prove that the sum of the squares of two consecutive even numbers is always divisible by 4.

(2) Prove that the difference between the cubes of any consecutive integers is always one more than a multiple of 3.

WORKING AT A*/A

(1) Prove, that if a and b are both positive numbers, then $\frac{a^2+b^2}{2ab} \geq 1$

(2) Prove, that if x and y are both positive integers, then $\frac{y}{x} + \frac{x}{y} \geq 2$