GCSE 9 -1
Mathematics
Higher Tier
Grade 9 'Tough Paper'
Paper 1

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CLICK ON THE QUESTION NUMBER FOR THE WORKED VIDEO SOLUTION

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(1) Given that $x(a+bx)(a-bx) \equiv 25x - 4x^3$, find the value of b^{-a}.

(2) Freda plays the lottery.
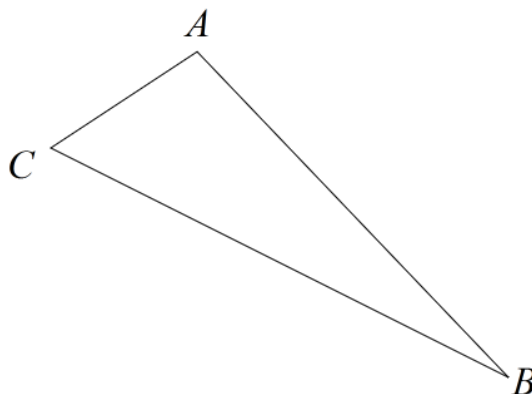There are 49 balls to choose from.
The balls are numbered 1 – 49.
Freda chooses the 6 numbers shown below in the order in which they appear.

John believes the numbers were chosen randomly.
Show that John could be wrong stating a reason for your choice.

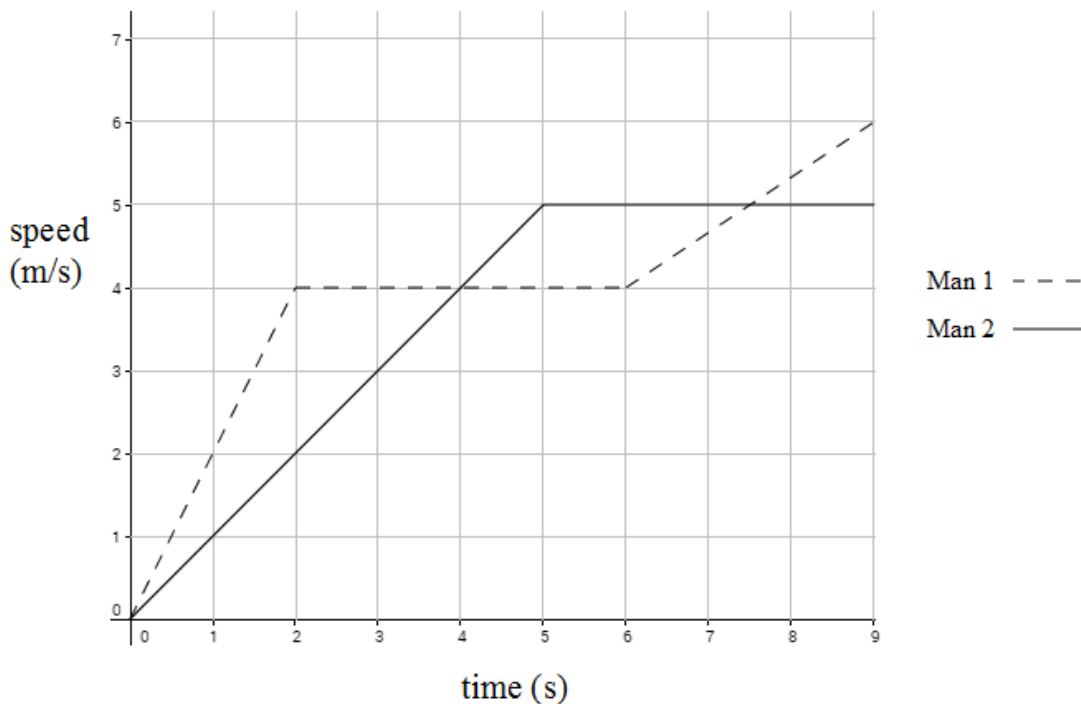(3) Triangle *ABC* is shown in the diagram below.

$AC = x$
$BC = 3x$
Angle $ACB = 60^o$
Show that the perimeter of the triangle is $\left(4+\sqrt{7}\right)x$.

(4) Find the value of $\left(\dfrac{1}{0.16}\right)^{1.5}$

(5) Two men walk together along a road, starting at the same time.
The speed-time graph below shows the first 9 seconds of the walk.

The ratio of the distance covered by Man 1 to the distance covered by Man 2 in the first 9 seconds of the walk can be written in the form $m:n$ where m and n are double digit integers.
Find the value of m and the value of n.

(6) Triangle ACD is shown in the diagram below.
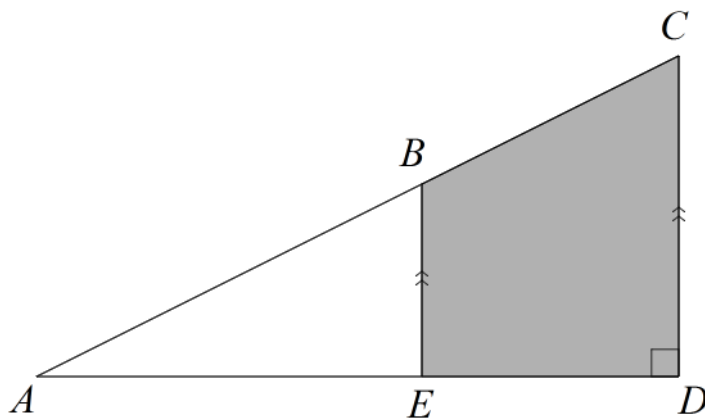
AED is a straight line.
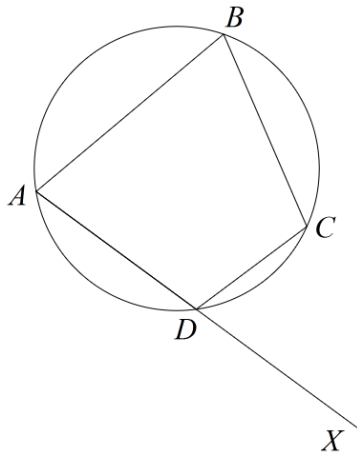$AB = 3\sqrt{5}$
$AE = 2BE$
$3AD = 5AE$
Find the area of the shaded quadrilateral $BCDE$.

<u>(7)</u> Find the value of p :

$$\left(\frac{\cos(60^o)}{\sin(60^o)} + \frac{10}{\sqrt{12}} \right)^2 = p$$

<u>(8)</u> A, B, C and D are all points on the circumference of a circle as shown in the diagram below.

Angle $DAB = x^2 - 5x - 8$
Angle $BCD = x^2 + 4x - 88$
Angle $CDA = y^2 - 15y + 90$
Angle $ABC = 5y - 6$
A line is drawn from D to X .
Angle $CDX = x^2 - 70$
Prove that ADX is a straight line.

<u>(9)</u> The first five terms of an arithmetic sequence are:

$$x+1, \quad 2x, \quad \frac{2(2x+3)}{6-x}, \quad x^2 - 2, \quad 5x - 3$$

Show that the term $4x^2 - 3$ is not in the sequence.

<u>(10)</u> A solid hemispherical piece of gold with diameter $1.2 \times 10^2 \, cm$ is to be melted into identical solid cuboids.
The dimensions of the cuboids are $30cm, 12cm$ and $10\pi cm$.
Find the number of cuboids that can be made from the hemisphere.
(You may assume there is no wastage in the production process.)

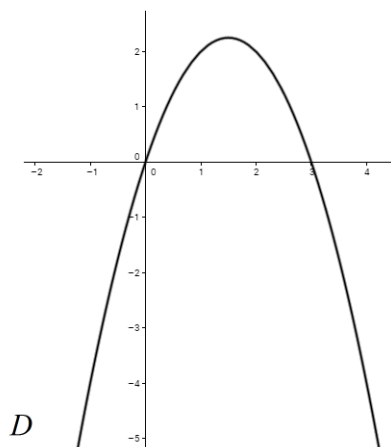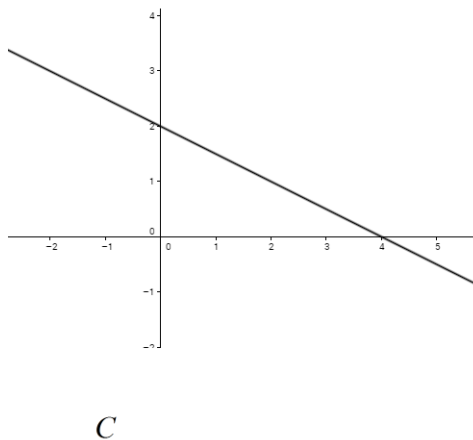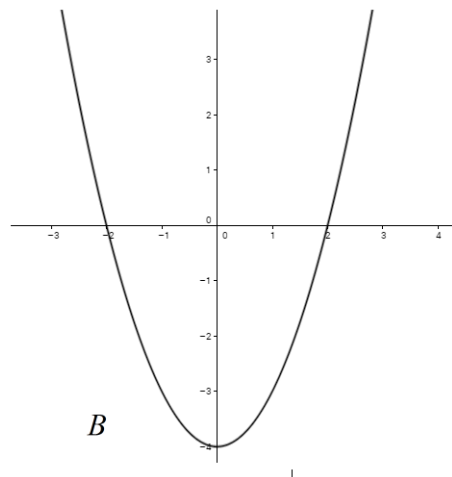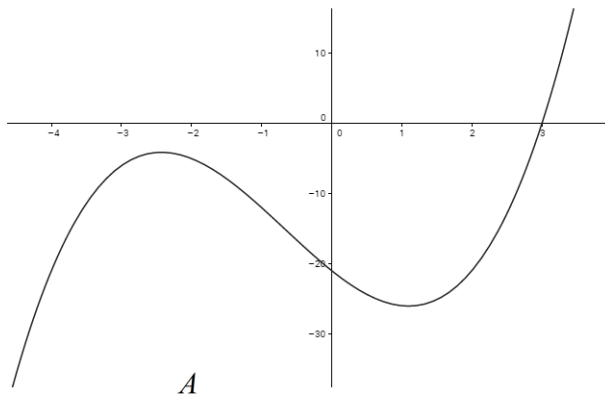(11) Two functions are given below:

$$f(x) = (x + p)(x + q)$$

$$g(x) = \frac{r}{x}, \quad x \neq 0$$

p, q and r are constants.

State which of the following graphs could be used to solve the equation $f(x) = g(x)$
You must give a reason for your choice.

A

B

C

D

(12) A is inversely proportion to $B^{\frac{1}{3}}$ and C is directly proportional to the square of B.
When $A = 0.5$, $B = 64$.
When $C = 15$, $B = 5$.
Express C in terms of A.

(13) Mr Lucky plays two games.
The two games are Game A and Game B.
Playing Game A and playing Game B are independent events.

The probability that Mr Lucky wins both games is $\dfrac{9}{25}$.

The probability that Mr Lucky wins Game B is four times greater than the probability of him losing Game A.
Find the probability that Mr Lucky wins only one of the two games he plays.
You must show full workings.

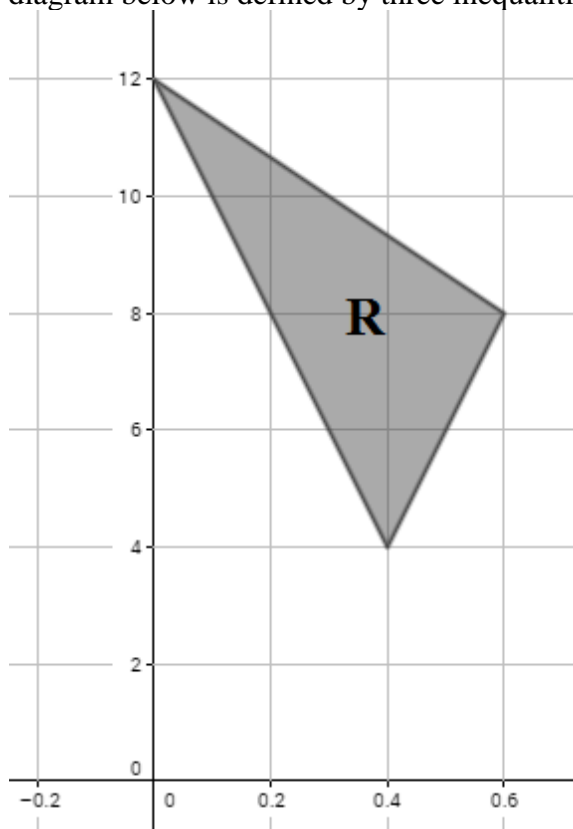(14) The diagram below shows the triangle ABC with a shaded square drawn inside.
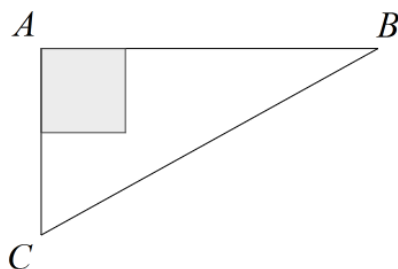
Triangle ABC is half of an equilateral triangle.
Angle $BAC = 90^o$
The shaded square touches the lines AB and AC and has side length $0.25AB$.
Show that the ratio of the area of the triangle to the area of the square is $8:\sqrt{3}$

(15) The region **R** shown in the diagram below is defined by three inequalities.

Find the three inequalities in the form $ay + bx \geq c$ where a, b and c are integers.
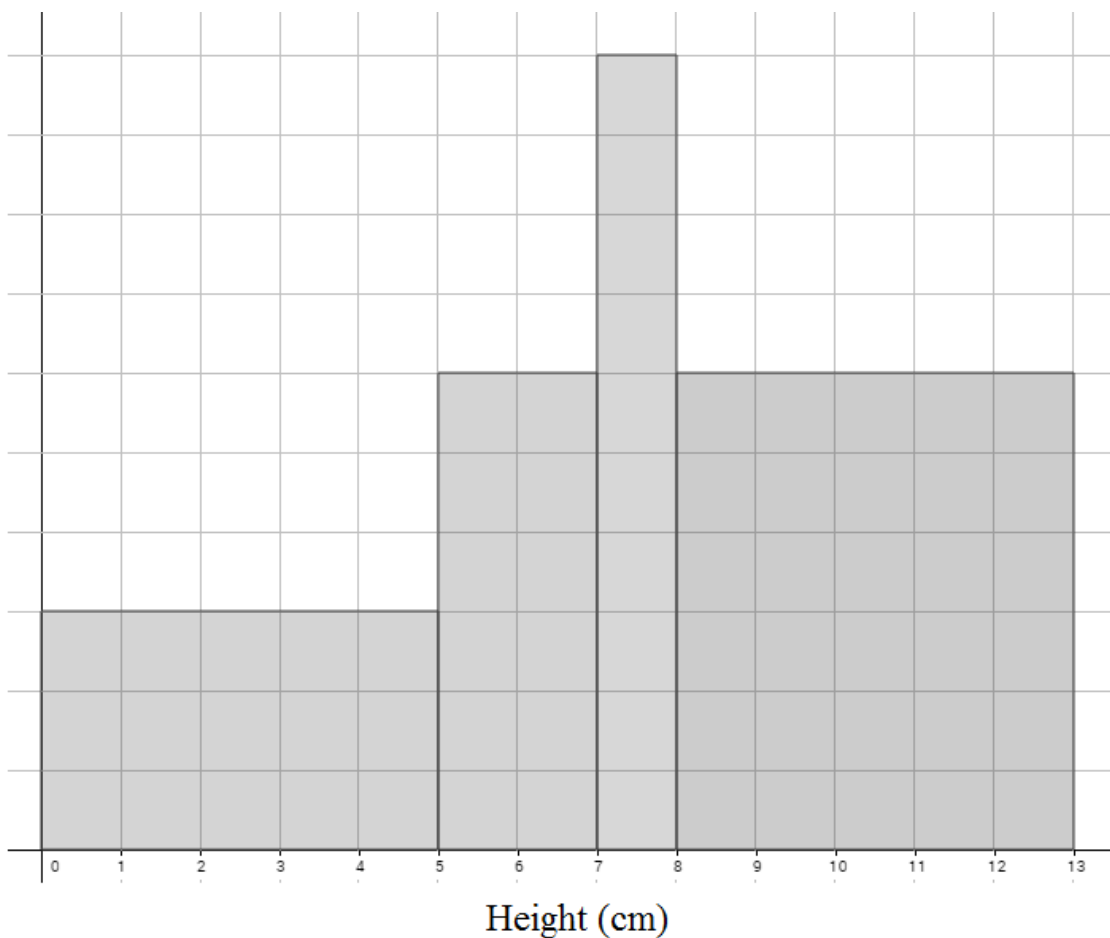
(16) Two vectors are defined as follows:

$$\overrightarrow{AC} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$\overrightarrow{AB} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$$

Find the value of $\cos(ACB)$ in its simplest form.

(17) The histogram below shows information about the height (cm) of a number of plants.
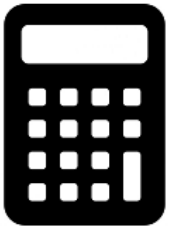
Height (cm)

There were 40 plants between 7 and 8cm tall.

Michael takes two plants at random from the sample and doesn't replace them.
He writes down his calculations for the probability and its answer as:

$$\frac{30}{67} \times \frac{16}{89} = \frac{480}{5963}.$$

Write down the minimum height of each of the plants Michael chooses.

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Paper 2

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<u>(1)</u> Island X is a small island.

In the winter of 1986 the ratio of natives to tourists on the island was 7:1.
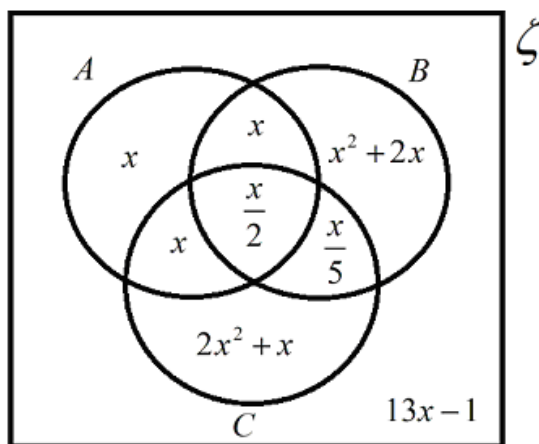
In the summer of 1987 the ratio of natives to tourists on the island was 155:69.

The number of natives on the island decreased by 100 from the winter of 1986 to summer of 1987.

The number of tourists on the island increased by 220 from the winter of 1986 to summer of 1987.
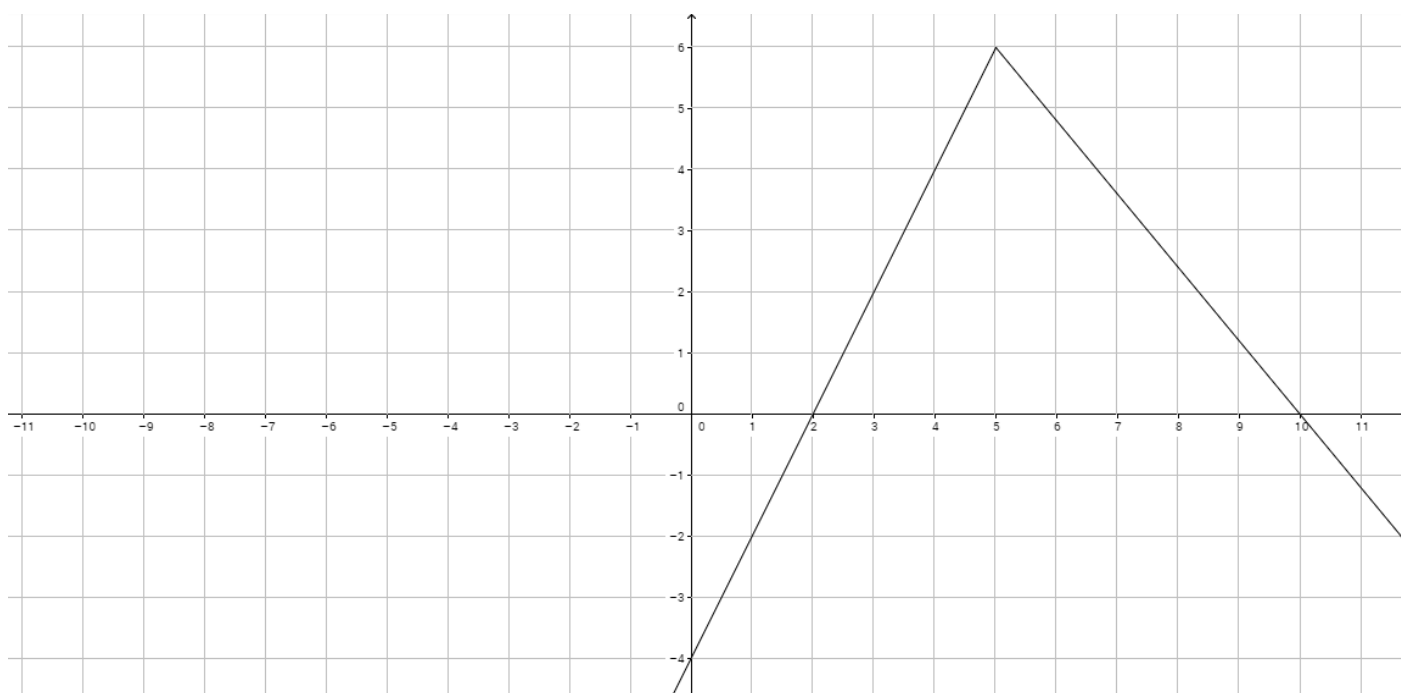
Find the number of tourists that were there on Island X in the winter of 1986.
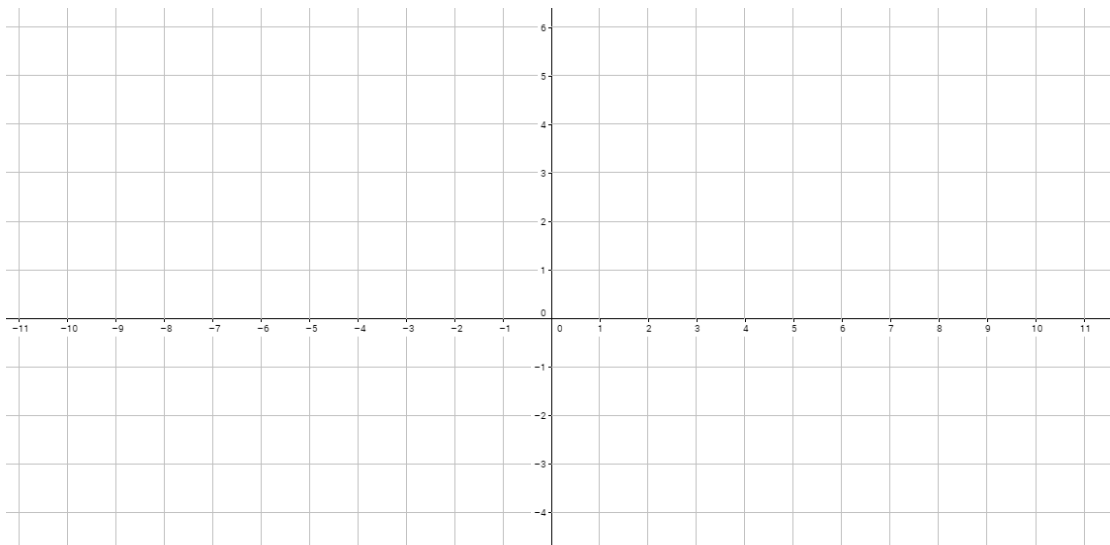
<u>(2)</u> Find $P((B \cap A')|C)$.

Give your answer as a fraction in the form $\dfrac{a}{b}$ where a and b are integers.

<u>(3)</u> The diagram below shows part of the graph of $y = 2f(x-1)$

On the grid below draw the graph of $y = -f(-x)$

(4) Using algebra, show that part of the line $3x + 4y = 0$ is a diameter of the circle with equation $x^2 + y^2 = 25$.

(5) Sue is making a toy rocket in her science lesson which is to be launched from the ground.
The flight path of the toy rocket can be modelled by the equation $h = -2t^2 + 6t + 1$.
h is the height in metres the rocket reaches above the ground.
t is the time in seconds after the rocket is launched.
Find the maximum height above the ground that the rocket reaches and the time it takes to reach this height.

(6) Shape A is a regular polygon.
The ratio of the size of the interior angle to the size of the exterior angle is 7:2.
The ratio of the side **length** to the **number** of sides is 5:3.
Find the perimeter of Shape A.

(7) Jamal is going to paddle between 3 points on a large lake.
Jamal will start from Point A and paddle on a bearing of 050^0 until he reaches Point B.
Once at Point B, Jamal will then paddle 1.6km on a bearing of 142^0 to reach Point C.
Finally Jamal will paddle directly back to Point A from Point C where he will finish.
The bearing of Point C from Point A is 098^0.
Given that Jamal can paddle at an average speed of 7.2kph, find the time it will take him to paddle directly back to Point A from Point C.
Give your answer to the nearest minute.

(8) Triangle ABC is an isosceles triangle such that $AB = BC$.
The points X and Y lie on the line AC.
$AY = 3AX$ and $AC = 4AX$
Prove that triangle ABX and triangle CBY are congruent.

(9) Company T are designing a toy to be sold online.
The toy will be made up of a hemisphere with radius Xcm and a right cone with radius Xcm and height Ycm.
The cone will be attached to the top of the hemisphere as shown below.
Given that the total mass of the toy is 100π grams and the density of the toy is $60\,g\,/\,cm^3$, express Y in terms of X. Give your answer in its simplest form.
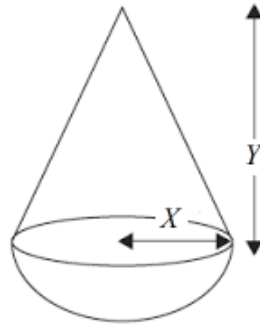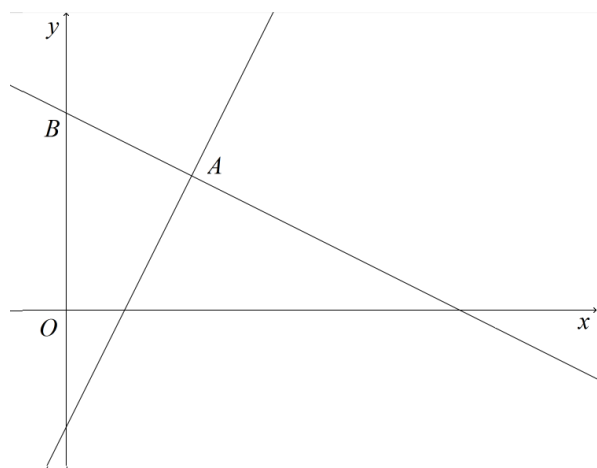

(10) There are N boys in a class at school.
For every 2 boys in the class there are 3 girls in the class.
3 students are chosen at random and taken out of the class.

Given that the probability of choosing 3 boys is $\dfrac{1}{30}$, show that $23N^2 - 114N + 88 = 0$

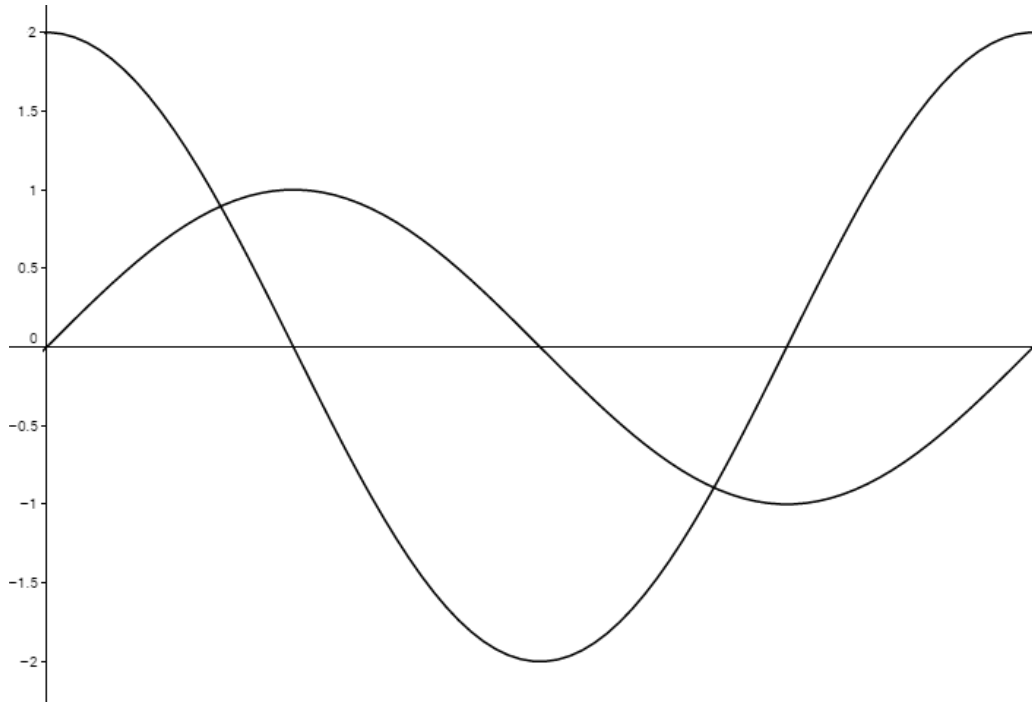(11) The diagram below shows Line 1 and Line 2.

Line 1 has gradient 2.
Line 2 is perpendicular to Line 1.
Line 1 and Line 2 intersect at the point $A(p,q)$.
Line 2 crosses the y axis at the point B.

Show that the coordinates of point B can be written as $\left(0, \dfrac{p}{2} + q\right)$.

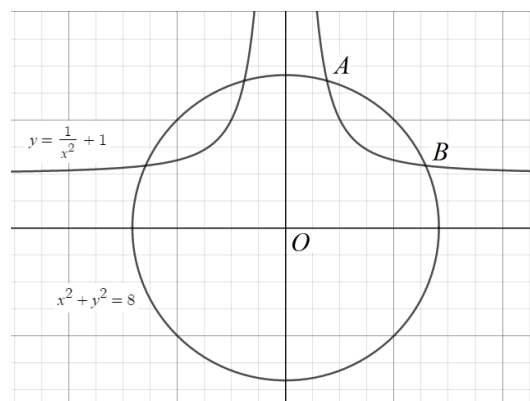<u>(12)</u> The graphs of $y = 2\cos(x)$ and $y = \sin(x)$ are shown in the diagram below for $0 \le x \le 360^o$.

Use the graphs to find estimates for the solutions of the equation:

$$\sin(x) - 2\cos(x) = 0 \text{ for } 0 \le x \le 360^o.$$

You must show all of your working.

<u>(13)</u> The diagram below shows the graph of the equation $x^2 + y^2 = 8$ and part of the graph of the equation $y = \dfrac{1}{x^2} + 1$.

The points $A(p,q)$ and $B(r,s)$ are two of the points where the graphs intersect.
(a) Using the graphs above, find the solutions to the simultaneous equations:

$$y = \frac{1}{x^2} + 1$$
$$x^2 + y^2 = 8$$

giving your answers in terms of p, q, r and s.

(b) There are 2 real solutions to the simultaneous equations: $x^2 + y^2 = 8$
   $x = a$

Find the set of values of a giving your answer in simplified surd form.

(14) The students in Class X and Class Y sat the same maths exam.
Information is given about the performance of each class in the table below.

	X	Y
Lowest Score	$x-1$	$y+1$
Lower Quartile	$x+2$	$2(y+1)$
Median	x^2-3	$y(y-1)$
Upper Quartile	$4x+2$	$3y+1$
Highest Score	$2(x^2+2)$	$5y-4$

The median score for Class X was half the median score for Class Y.
The interquartile range for Class X was three times the interquartile range for Class Y.
Michael scored 17 marks in his maths exam.
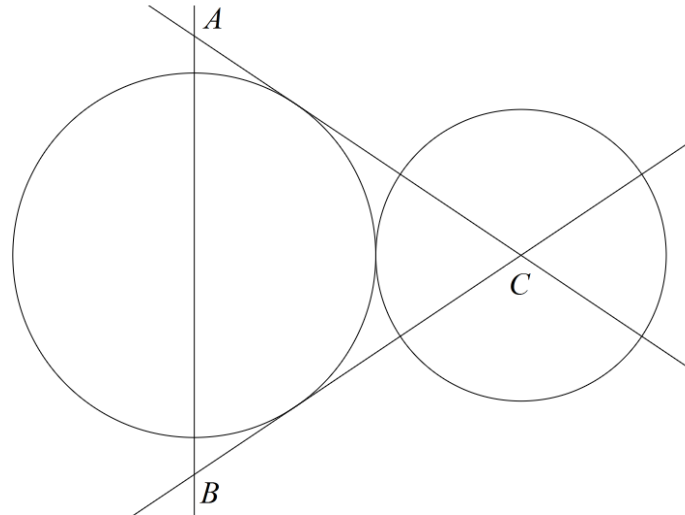Complete the following sentence;   *"Michael was in the top_____% of performers in Class___"*

You must show all of your working.

(15) The diagram below shows parts of the graphs of $y = f(x)$ and $y = g(x)$.

$y = f(x)$ $y = g(x)$
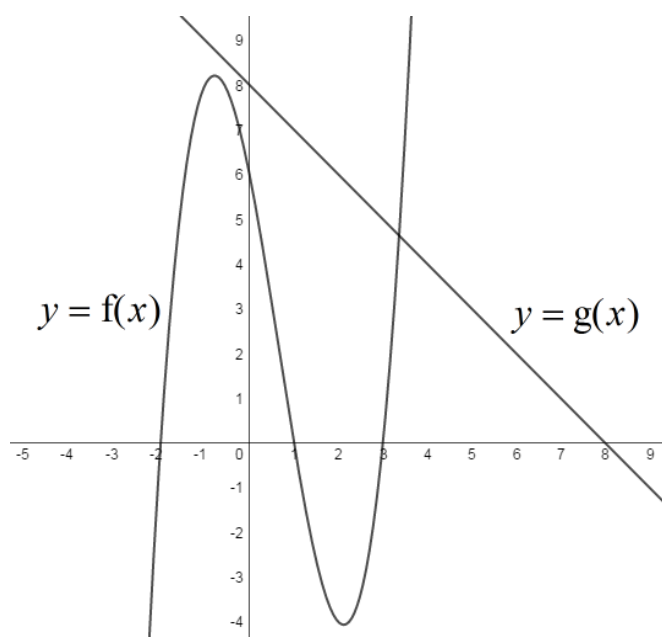
Find the integer value of $gff(-2)$.

(16) The diagram below shows two touching circles, Circle 1 and Circle 2.

Circle 1 has radius 5cm.
Circle 2 has radius 4cm.
C is the centre of the Circle 2.
The line AB lies on a diameter of Circle 1 .
The line AB is perpendicular to a line passing through the centre of both circles.
AC and BC are tangents to Circle 1 and pass through C .
Find the area of triangle ABC .
Give your answer to 3 significant figures.

(17) The area of a parallelogram is $15cm^2$ correct to the nearest integer.
The shortest side of the parallelogram is $4.5cm$ correct to 2 significant figures.
The longest side of the parallelogram is $7.1cm$ correct to 2 significant figures.
Find the largest possible size of the two acute angles in the parallelogram.
Give your answer correct to 3 decimal places.