Differentiation
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(1) $y = x^2 + 4x$

Show that when the rate of change of $y = 8$

$$2(x-2) = 0$$

(2) Given that

$$y = x^3\left(4x^2 + x - \frac{5}{x}\right)$$

Show that

$$\frac{dy}{dx} = 2x(10x^3 + 2x^2 - 5)$$

(3) A curve has the equation

$y = \frac{2}{3}x^3 - 2.5x^2 - 3x + 4$

Show that the curve is stationary when $x = -0.5$ and when $x = 3$.

(4) $y = \frac{4x^6 - 3x^2 - 6}{x^2}$

Find a simplified expression for $\frac{dy}{dx}$.

(5) A curve has equation

$y = \sqrt{x} - 8x$

Show that the curve is stationary when $x = \frac{1}{256}$

(6) $y = (x^{\frac{2}{3}} - 4)(x^{\frac{1}{5}} + 2)$

Find a simplified expression for $\frac{dy}{dx}$.

(7) A curve has equation

$y = 0.25x^4 - \frac{8}{3}x^3 + 6x^2$

(a) Show that the curve has 3 stationary points.

(b) Find the coordinates of the stationary points.

(c) Determine the nature of each of the stationary points.

(8) A curve has equation

$y = (x^{\frac{1}{2}} - 3)^2$

Show that $\frac{dy}{dx} = 1 + Ax^n$

stating the values of A and n.

(9) A cubic curve has equation

$y = \frac{10}{3}x^3 + \frac{1}{2}x^2 - 6x + 2$

Find the coordinates of the two stationary points on the curve and determine their nature. Give each coordinate to 3SF.

(10) A curve has equation

$y = \frac{1}{\sqrt[3]{x}} + 27x$

(a) Find the x coordinate of the only stationary point on the curve.

(b) Determine the nature of the stationary point.

① $y = x^2 + 4x$

$\frac{dy}{dx} = 2x + 4$

$8 = 2x + 4$

$0 = 2x - 4$

$0 = 2(x - 2)$

② $y = 4x^5 + x^4 - 5x^2$

$\frac{dy}{dx} = 20x^4 + 4x^3 - 10x$

$= 2x(10x^3 + 2x^2 - 5)$

③ $\frac{dy}{dx} = 2x^2 - 5x - 3$

SP=0 for $\frac{dy}{dx}$

$\therefore\ 2x^2 - 5x - 3 = 0$

$(2x+1)(x-3) = 0$

$x = 3,\ x = -\frac{1}{2}$

④ $y = \frac{4x^6}{x^2} - \frac{3x^2}{x^2} - \frac{6}{x^2}$

$y = 4x^4 - 3 - 6x^{-2}$

$\frac{dy}{dx} = 16x^3 + 0 + 12x^{-3}$

$= 16x^3 + 12x^{-3}$

or $16x^3 + \frac{12}{x^3}$

⑤ $y = x^{\frac{1}{2}} - 8x$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 8$

$0 = \frac{1}{2}x^{-\frac{1}{2}} - 8$

$8 = \frac{1}{2}x^{-\frac{1}{2}}$

$16 = x^{-\frac{1}{2}}$

$\frac{1}{16} = x^{\frac{1}{2}}$

$\left(\frac{1}{16}\right)^2 = x$

$\frac{1}{256} = x$

⑥ $y = x^{13/15} + 2x^{\frac{2}{3}} - 4x^{\frac{1}{5}} - 8$

$\frac{dy}{dx} = \frac{13}{15}x^{-\frac{2}{15}} + \frac{4}{3}x^{-\frac{1}{3}} - \frac{4}{5}x^{-\frac{4}{5}}$

⑦ $y = \frac{1}{4}x^4 - \frac{8}{3}x^3 + 6x^2$

$\frac{dy}{dx} = x^3 - 8x^2 + 12x$

$0 = x(x^2 - 8x + 12)$

$0 = x(x-6)(x-2)$

SP: $x = 0,\ x = 6\ x = 2$

(a)

(b) when $x=0$ $y=0$ $\therefore (0,0)$

when $x=6$ $y=-36$ $\therefore (6,-36)$

when $x=2$ $y=\frac{20}{3}$ $\therefore \left(2, \frac{20}{3}\right)$

(c) $\frac{d^2y}{dx^2} = 3x^2 - 16x + 12$

when $x=0$ $\frac{d^2y}{dx^2} = 12$

$\therefore$ min

when $x=6$ $\frac{d^2y}{dx^2} = 24$

$\therefore$ min

when $x=2$ $\frac{d^2y}{dx^2} = -8$

$\therefore$ max

⑧ $y = (x^{\frac{1}{2}} - 3)(x^{\frac{1}{2}} - 3)$

$y = x - 6x^{\frac{1}{2}} + 9$

$\frac{dy}{dx} = 1 - 3x^{-\frac{1}{2}}$

$A = -3\quad n = -\frac{1}{2}$

⑨ $\frac{dy}{dx} = 10x^2 + x - 6$

$10x^2 + x - 6 = 0$

$x = \frac{-1 + \sqrt{241}}{20}$

$x = \frac{-1 - \sqrt{241}}{20}$

when $x = \frac{-1+\sqrt{241}}{20}$

$y = -0.8169$

$\therefore \left(\frac{-1+\sqrt{241}}{20}, -0.817\right)$

when $x = \frac{-1-\sqrt{241}}{20}$

$y = 5.42$

$\therefore \left(\frac{-1-\sqrt{241}}{20}, 5.42\right)$

$\frac{d^2y}{dx^2} = 20x + 1$

when $x = \frac{-1+\sqrt{241}}{20}$

$\frac{d^2y}{dx^2} > 0\ \therefore$ min

when $x = \frac{-1-\sqrt{241}}{20}$

$\frac{d^2y}{dx^2} < 0\ \therefore$ max

⑩ $y = x^{-\frac{1}{3}} + 27x$

$\frac{dy}{dx} = -\frac{1}{3}x^{-\frac{4}{3}} + 27$

$0 = -\frac{1}{3}x^{-4/3} + 27$

$x^{-4/3} = \frac{1}{81}$

$x = \frac{1}{81}^{3/4}$ or $\frac{1}{27}$

$\frac{d^2y}{dx^2} = \frac{4}{9}x^{-7/3}$

$\therefore$ min as $\frac{d^2y}{dx^2} > 0$ for $x = \left(\frac{1}{27}\right)^{-7/3}$