Core Pure 1 – Chapters 1-4 Test 1 – www.m4ths.com – Steve Blades ©

(1) Write $4 - 4i$ in the form $r(\cos\theta + i\sin\theta)$

(2) the complex numbers z_1 and z_2 are such that $z_1 = -3 + 4i$ and $|z_1 z_2| = 10$. Given further that $\arg(z_2) = -\pi$,
(a) Express $z_1 - z_2$ in the form $a + bi$.
(b) Plot $z_1 - z_2$ on an Argand diagram.

(3) The complex numbers $a + bi$ and $c + di$ lie in the region which satisfies:

$$\{z\epsilon C: |z - 4 - 3i| \leq 4\} \cap \left\{z\epsilon C: \frac{\pi}{4} \leq \arg(z - 4 - 3i) \leq \frac{3\pi}{4}\right\}$$

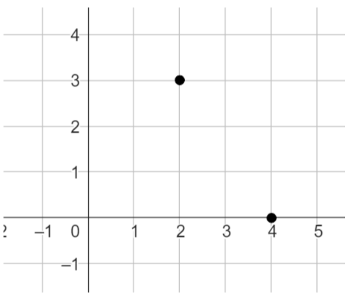
Find the maximum value of $c - a$ in the form $p\sqrt{q}$.

(4) A cubic equation has the roots α, β and γ.
Given that $\sum\alpha = 6, \sum\alpha\beta = 25$ and $\alpha\beta\gamma = 82$, find:
(a) $\alpha^2\beta^2\gamma^2$
(b) $\alpha^2 + \beta^2 + \gamma^2$
(c) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$
(d) An equation with integer coefficients that has the roots $\alpha - 3$, $\beta - 3$ and $\gamma - 3$.

(5) Find possible values of a, b, c, d and e for the sum given below

$$\sum_{r=2n+1}^{4n} r^2 = an(bn + c)(dn + e)$$

(6) A quartic equation has roots α, β, γ and δ. The two points shown on the Argand diagram below represent roots of the quartic equation.



Given that the quartic equation can be written in the form $(x - p)^2(x^2 + qx + r) = 0$, show that $\alpha\beta\gamma\delta = 208$.

(7) Show that

$$\sum_{r=1}^{n} 12r(r + 1)(r - 1) = 3n(n + 2)(n^2 - 1)$$