

Completing the Square for $x^2 + bx + c$

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Starter

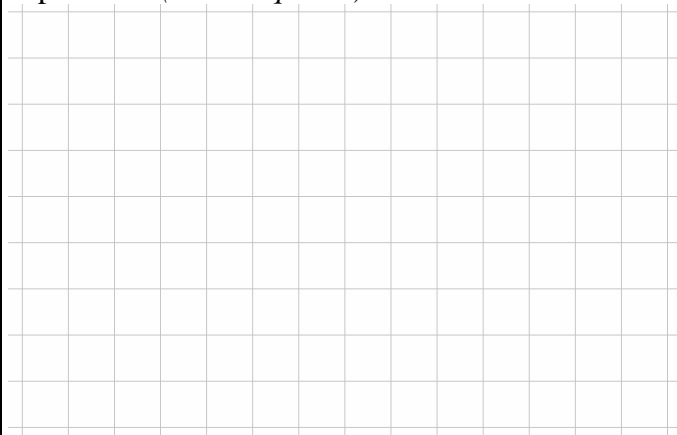
(1) Factorise the following quadratic expressions:

- (a) $x^2 + x - 12$
- (b) $x^2 - 2x - 8$
- (c) $x^2 - 5x - 6$
- (d) $x^2 - 10x + 25$
- (e) $x^2 + 4x + 4$
- (f) $x^2 + 12x + 36$

(2) What do you notice about the answers to question (d) to (f) above?

(3) (a) Factorise the expression $x^2 + 6x + 9$.

(b) Use the square paper below to draw a geometrical representation of your factorised expression. (*Think Square!*)



(4) (a) Factorise the expression $x^2 + 8x + 16$.

(b) Use the square paper below to draw a geometrical representation of your factorised expression.



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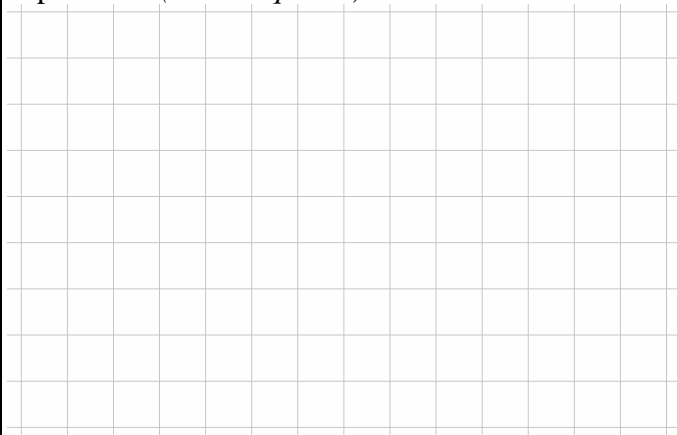
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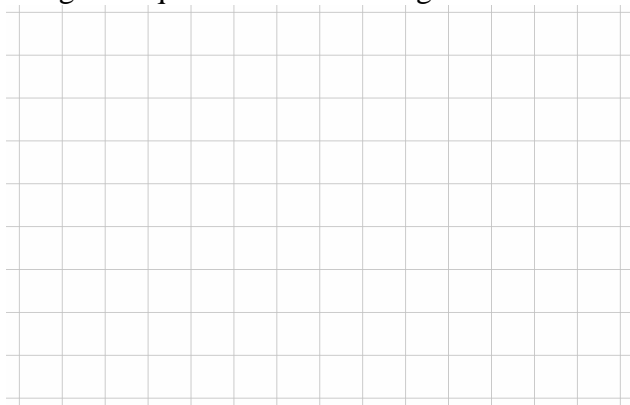
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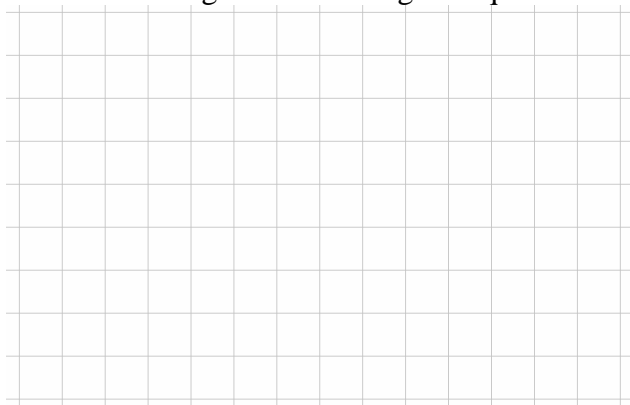
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(1) (a) Draw a geometrical representation of $x^2 + 6x$ using one square and one rectangle below.



(b) Split the rectangle into two new equal sized rectangles and try and make a larger square with the two new rectangles and the original square.



(c) What would you need to add complete the square?

(d) Add this to your diagram below.



(e) Based on the diagrams you have drawn; explain how you would complete square for the quadratic expression $x^2 + 6x$ and the general form $x^2 + bx$.

(f) How could you explain this as a set of instructions?

(g) Write an algebraic expression for the completed square form of the quadratic $x^2 + bx$?

(2) Write the following expressions in the form $(x + a)^2 + b$ leaving your values of a and b as fractions where appropriate.

(a) $x^2 + 4x$

(b) $x^2 - 8x$

(c) $x^2 + 2x$

(d) $x^2 + 5x$

(e) $x^2 - 7x$

(f) $x^2 + 6x + 1$

(g) $x^2 - 12x - 3$

(h) $x^2 - \frac{1}{2}x + 3$

(i) $2x + 5 + x^2$

(j) $x^2 + tx$

(k) $x^2 + tx + t^2$ (where t is a constant)

(l) $3tx + x^2 - t^2$

Completing the Square - Extension

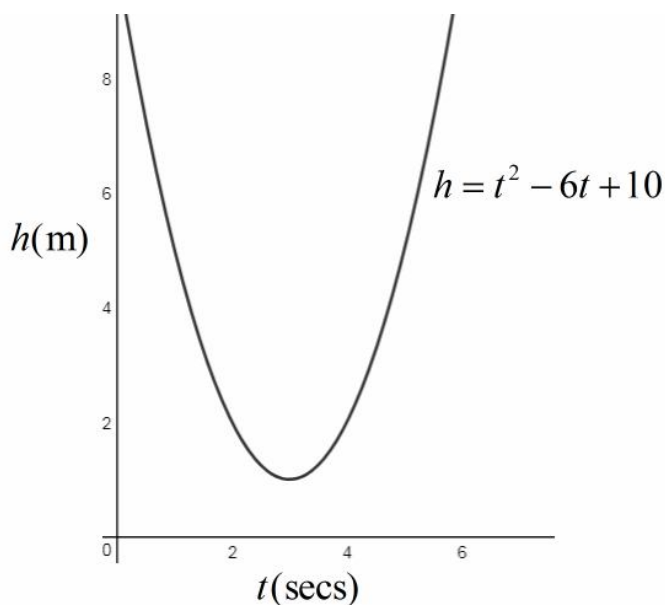
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(1) Draw a geometrical representation to show a student how $x^2 - 4x$ can be written in completed square form.



(2) The completed square form can be used to find maximum and minimum points on the graphs of quadratic functions.

The diagram below shows part of a graph representing the height of a ball above the ground.



(a) What is the closest the ball will get to the ground and how long will it take it to reach this point? (*You must use algebra to show this!*).

(b) Explain what graph transformations have taken place to transform the graph of $h = t^2$ to the graph of $h = t^2 - 6t + 10$

(3) The quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

is used to find the solutions to the equation $ax^2 + bx + c = 0$.

The formula can be derived by completing the square.

John has done the first step of the derivation below:

$$a \left(x^2 + \frac{b}{a}x + \frac{c}{a} \right) = 0$$

Continue the derivation below by completing the square:

Solving Quadratic Equations using the Completed Square method.

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(1) Solve each of the following quadratic equations using the method of completing the square. Check each answer by factorising where possible.

(a) $x^2 + 4x - 12 = 0$

(b) $x^2 - 2x - 8 = 0$

(c) $x^2 + 3x - 10 = 0$

(d) $x^2 - 5x - 14 = 0$

(e) $2x^2 - 2x - 24 = 0$

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