Algebraic Proof www.m4ths.com

① $n^2+10n+25-n^2 \equiv 10n+25$
$\equiv 5(2n+5)$ ✓

② $(\cancel{4n^2}+4n+\cancel{1})-(\cancel{4n^2}-4n+\cancel{1})$
$\equiv 4n--4n$
$\equiv 8n$ ✓

③ $n^2+6n+9+n^2-5 \equiv 2n^2+6n+4$
$\equiv 2(n^2+3n+2)$
$\equiv 2(n+1)(n+2)$ ✓

④ $(\cancel{n^2}+6n+\cancel{9})-(\cancel{n^2}-6n+\cancel{9}) \equiv$
$\equiv 6n--6n$
$\equiv 12n$ ∴ a multiple of 12

⑤ $(n^2+6n+9)+(n^2+10n+25)$
$\equiv 2n^2+16n+34$
$\equiv 2(n^2+8n+17)$
∴ always even as it's a multiple of 2. (or divisible by 2)

⑥ Common factor of $(n-4)^2$
$(n-4)^2(1+(n-4))$
$\equiv (n-4)^2(n-3)$
$\equiv (n-3)(n-4)^2$ ✓

⑦ Complete the square
$(n-4)^2-16+18$
$\equiv (n-4)^2+2$
The least value of $(n-4)^2$ is 0
∴ the least value of $(n-4)^2+2$ is 2 which is >0.

⑧ $(4+12n+9n^2)+(16+8n+n^2)$
$\equiv 20+20n+10n^2$
$\equiv 10(2+2n+n^2)$ ✓
∴ divisible by 10

⑨ An odd number can be written as $2n+1$.
Expanding
n^2+n^2+2n+1
$\equiv 2n^2+2n+1$
$\equiv 2(n^2+n)+1$
∴ odd as $2(n^2+n)$ is always even so $2(n^2+n)+1$ will always be odd.

⑩ Let the integers be n and $n+1$
$n+n+1 \equiv 2n+1$
$2n$ is always even ∴ $2n+1$ is always odd ✓

(11) let the two consecutive odd integers be $2n-1$ and $2n+1$.

$\therefore (2n+1)-(2n-1)$

$\equiv 1 - -1$

$\equiv 2$ ✓

(12) let the consecutive even integers be $2n-2, 2n$ and $2n+2$.

$(2n\cancel{-2})+(2n)+(2n\cancel{+2}) \equiv 6n$ ✓

$\therefore$ a multiple of 6.

(13) let the consecutive even integers be $2n$ and $2n+2$

$2n \times 2n+2$

$\equiv 2n \times 2(n+1)$

$\equiv 4n(n+1)$ ✓

$\therefore$ always a multiple of 4.

(14) let the consecutive integers be $(n-1)$, n and $(n+1)$.

$(n-1)^2+n^2+(n+1)^2$

$\equiv n^2 \cancel{-2n}+1+n^2+n^2\cancel{+2n}+1$

$\equiv 3n^2+2$

$\equiv 3n^2+3-1$ ✓ $\therefore$ one less than a multiple of 3.

$\equiv 3(n^2+1)-1$

(15) $-4n^2+8n-5$

Complete the square

$-4[n^2-2n]-5$

$-4[(n-1)^2-1]-5$

$-4(n-1)^2+4-5$

$-4(n-1)^2-1$

We know $-4(n-1)^2<0$ for all values of n $\therefore -4(n-1)^2-1<0$ too.

(or the max value of the expression is -1 which is <0).

(16) let the numbers be $2n-1$ and $2n+1$

$(2n+1)^3+(2n-1)^3$

Binomial Expansion.

$\equiv 8n^3+3(2n)^2(1)+3(2n)(1)+1$

$+8n^3+3(2n)^2(-1)+3(2n)(-1)^2+(-1)^3$

$\equiv 16n^3+12n$

$\equiv 4[4n^3+3]$

$\therefore$ divisible by 4 ✓

(17) $2n+2$ and $2n$ are consecutive even integers

$(2n+2)^3-(2n)^3$

$\equiv \cancel{8n^3}+3(2n)^2(2)+3(2n)(2)^2+8\cancel{-8n^3}$

$\equiv 24n^2+24n+8$

$\equiv 8[3n^2+3n+1]$ $\therefore$ a multiple of 8. The least multiple of 8

$\therefore 8[3n^2+3n+1] \geq 8$.