Argand Diagrams Test – Further A Level Maths - www.m4ths.com - Steve Blades ©

(1) The complex number w is such that $w = 3 - 3\sqrt{3}\, i$

(a) Write down w^* in the form $x + iy$

(b) Find the modulus of w

(c) Find the argument of w

(d) Plot w and w^* on an Argand diagram

(e) Show that ww^* is a real number

(2) $z_1 = -2 + 6i$ and $z_2 = 3 - i$.

On an Argand diagram represent the following with vectors:

(a) $z_1 + z_2$

(b) $z_1 - z_2$

(3) Write $\frac{12\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)}{2\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)}$ in the form $x + iy$

(4) z is a complex number that can be written in the form $x + iy$. Given that $|z^2| = 8$ and $\arg(z^2) = \frac{\pi}{3}$, find the values of x and y.

(5) (a) Sketch each of the following loci on separate diagrams:

(i) $|z - 2 + 4i| = 3$ (ii) $|z + 2i| = |z + 2 - 6i|$ (iii) $\arg z - 3 - i) = \frac{-3\pi}{4}$

(b) Find the cartesian equation of each of the loci above.

(6) Given that $|w - 10 - 6i| = 6$, find:

(a) The least value of $|w|$ giving your answer in exact form.

(b) The greatest value of $\arg(w)$ giving your answer in the form $\theta = p \arctan q$ where p and q are rational numbers.

(7) Shade in an Argand diagram the set of points

$$\{z \in C: |z + 2| < 4\} \cap \left\{z \in C: -\frac{\pi}{4} \leq \arg(z - 1) < \frac{\pi}{4}\right\}$$

(8) $|z - 3 - i| = |z + 1 - 5i|$

(a) Sketch the locus of z

(b) Hence shade the region that satisfies $|z - 3 - i| > |z + 1 - 5i|$

(c) Find the least value of $|z|$

(9) The complex number w is such that $|w| > k$ where k is a real constant and $\frac{-\pi}{2} \leq \arg(w - k) \leq \frac{\pi}{4}$.
How many values does w have?

(10) $z_1 = 2p - 3pi$ and $z_2 = p + 2\sqrt{3}pi$ where p is a real positive constant. Given that $|z_1 z_2| = 52$, find the value of p.