Argand Diagrams Test – Further A Level Maths - www.m4ths.com - Steve Blades ©

Answers

(1) The complex number w is such that $w = 3 - 3\sqrt{3}\,i$

(a) Write down w^* in the form $x + iy$

(b) Find the modulus of w

(c) Find the argument of w

(d) Plot w and w^* on an Argand diagram

(e) Show that ww^* is a real number

(2) $z_1 = -2 + 6i$ and $z_2 = 3 - i$.

On an Argand diagram represent the following with vectors:

(a) $z_1 + z_2$

(b) $z_1 - z_2$

(3) Write $\frac{12\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)}{2\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)}$ in the form $x + iy$

(4) z is a complex number that can be written in the form $x + iy$. Given that $|z^2| = 8$ and $\arg(z^2) = \frac{\pi}{3}$, find the values of x and y.

(5) (a) Sketch each of the following loci on separate diagrams:

(i) $|z - 2 + 4i| = 3$ (ii) $|z + 2i| = |z + 2 - 6i|$ (iii) $\arg z - 3 - i) = \frac{-3\pi}{4}$

(b) Find the cartesian equation of each of the loci above.

(6) Given that $|w - 10 - 6i| = 6$, find:

(a) The least value of $|w|$ giving your answer in exact form.

(b) The greatest value of $\arg(w)$ giving your answer in the form $\theta = p \arctan q$ where p and q are rational numbers.

(7) Shade in an Argand diagram the set of points

$$\{z \in C: |z + 2| < 4\} \cap \left\{z \in C: -\frac{\pi}{4} \le \arg(z - 1) < \frac{\pi}{4}\right\}$$

(8) $|z - 3 - i| = |z + 1 - 5i|$

(a) Sketch the locus of z

(b) Hence shade the region that satisfies $|z - 3 - i| > |z + 1 - 5i|$

(c) Find the least value of $|z|$

(9) The complex number w is such that $|w| > k$ where k is a real constant and $\frac{-\pi}{2} \le \arg(w - k) \le \frac{\pi}{4}$.
How many values does w have?

(10) $z_1 = 2p - 3pi$ and $z_2 = p + 2\sqrt{3}pi$ where p is a real positive constant. Given that $|z_1 z_2| = 52$, find the value of p.

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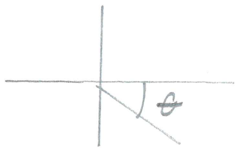
1 (a) $w^* = 3+3\sqrt{3}i$

(b) $|w| = \sqrt{3^2+(3\sqrt{3})^2}$

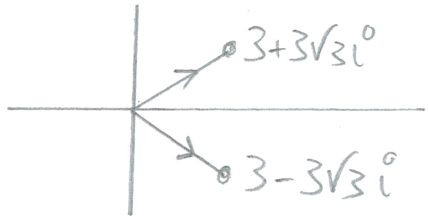
$= \sqrt{9+27}$

$= 6$

(c) 

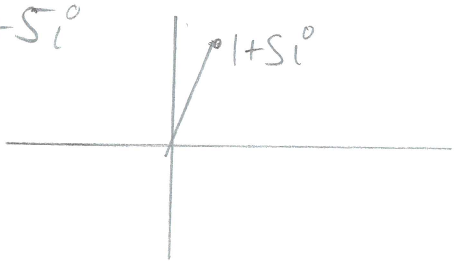
$\arctan\left(\frac{3\sqrt{3}}{3}\right) = \theta$

$\frac{\pi}{3} = \theta$ $\therefore \arg(z) = -\frac{\pi}{3}$

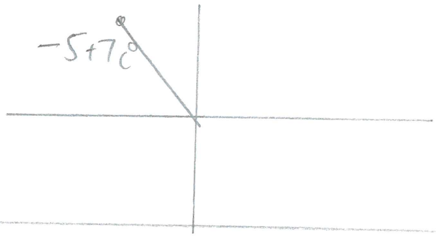
(d) 

(e) $(3+3\sqrt{3}i)(3-3\sqrt{3}i) = 9+9\sqrt{3}i-9\sqrt{3}i-27i^2 = 9+27 = 36$

2 (a) $z_1+z_2 = 1+5i$



(b) $z_1-z_2 = -5+7i$



3 Using $\frac{|z_1|}{|z_2|} = \left|\frac{z_1}{z_2}\right|$ $\frac{12}{2} = 6$ AND Using $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1)-\arg(z_2)$

$\frac{\pi}{2} - -\frac{\pi}{4} = \frac{3\pi}{4}$

$\therefore 6\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)$

$6\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right) = -3\sqrt{2}+3\sqrt{2}i$

4 Using $|z_1z_2| = |z_1||z_2|$ AND $\arg(z_1z_2) = \arg(z_1)+\arg(z_2)$

$|z||z| = 8$ $\therefore |z| = 2\sqrt{2}$

$\arg(z+z) = \frac{\pi}{3}$ $\therefore \arg(z)+\arg(z) = \frac{\pi}{3}$

$2\arg(z) = \frac{\pi}{3}$

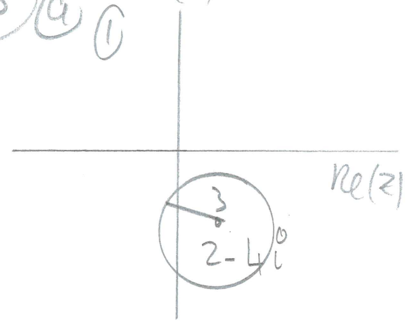
$\arg(z) = \frac{\pi}{6}$

$z = 2\sqrt{2}\left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)$

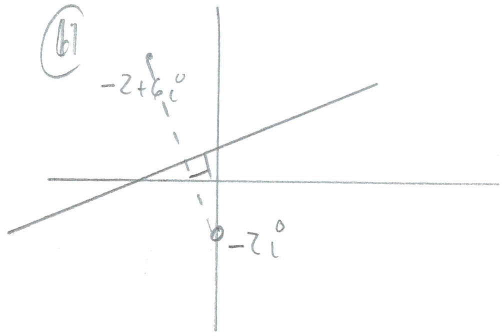
$= 2\sqrt{2}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$

$= \sqrt{6}+\sqrt{2}i$

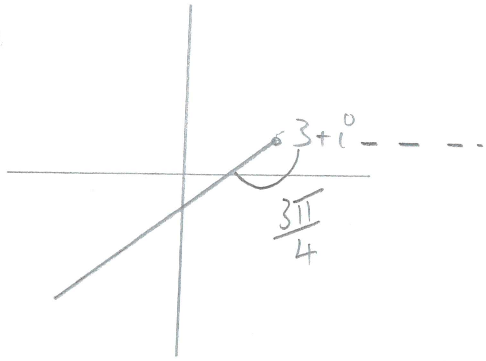
5 (a) (i)



(ii)



(iii)



(b) $(x-2)^2+(y+4)^2=9$

Midpoint $(-1,2)$

Gradient of original dotted line $= \frac{6--2}{-2-0} = -4$

$\therefore$ gradient of $\perp$ bisector $= \frac{1}{4}$

Line : $y-2=\frac{1}{4}(x+1)$ o.e.

$\frac{y-1}{x-3} = \tan\left(-\frac{3\pi}{4}\right)$

$\frac{y-1}{x-3} = 1$

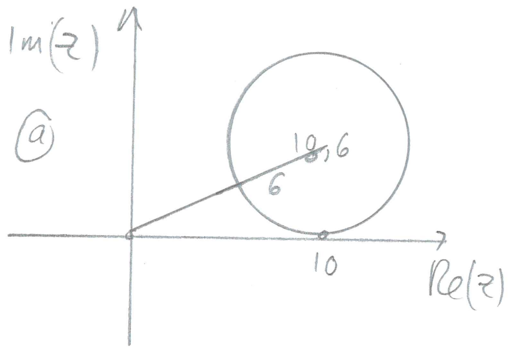
$y-1=x-3$

$y=x-2$

$x \in \mathbb{R},\ x<3$

6

(a)

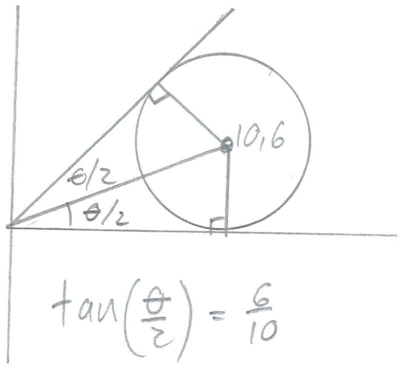


(a) $\sqrt{10^2+6^2}-6$

$=\sqrt{136}-6$

$=2\sqrt{34}-6$

(b)

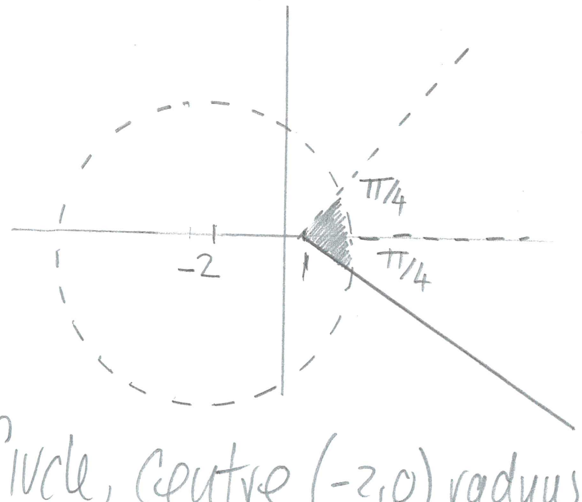


$\tan\left(\frac{\theta}{2}\right) = \frac{6}{10}$

$\frac{\theta}{2} = \arctan\left(\frac{6}{10}\right)$

$\theta = 2\arctan\left(\frac{3}{5}\right)$

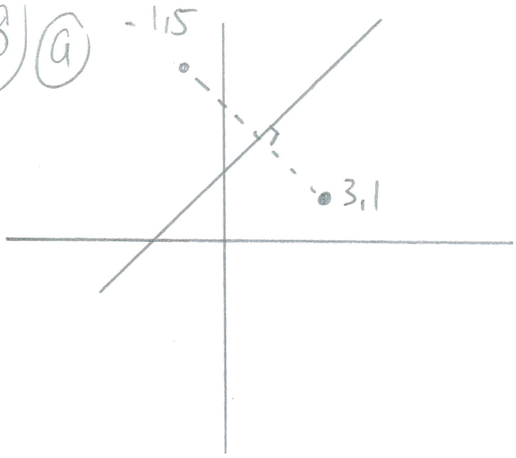
7



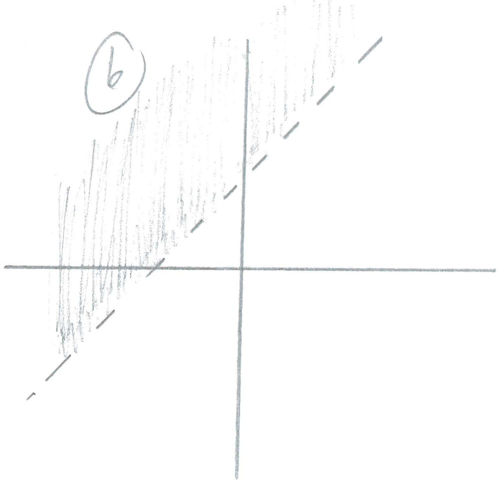
Half lines starting $(1,0)$

Circle, centre $(-2,0)$ radius 4

8 a



b



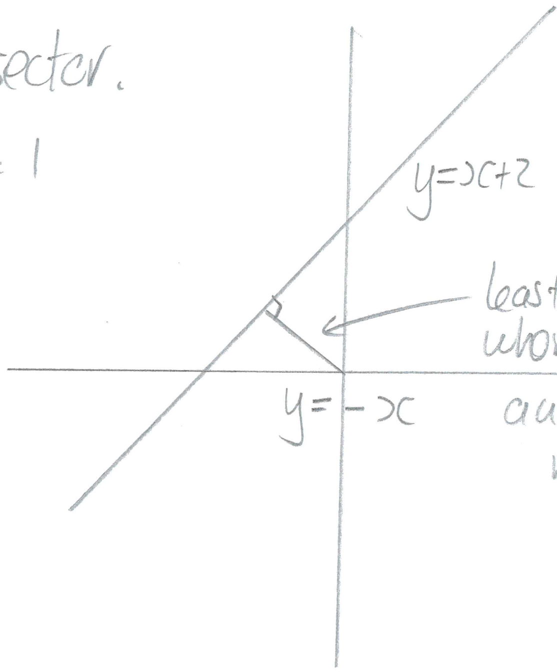
c Equation of the $\perp$ bisecter.

$m_1 = \frac{5-1}{-1-3} = \frac{-4}{4} = -1 \quad \therefore m_2 = 1$

Midpoint: 1,3

$\therefore y-3 = 1(x-1)$

$\underline{y = x+2}$



least distance where $y=x+2$ and $y=-x$ meat.

Simultaneous Equations

$y = x+2$

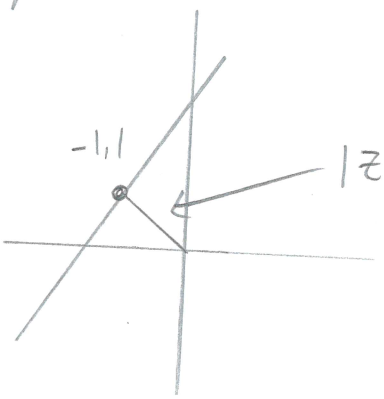
$y = -x$

$\therefore -x = x+2$

$-2x = 2$

$x = -1$

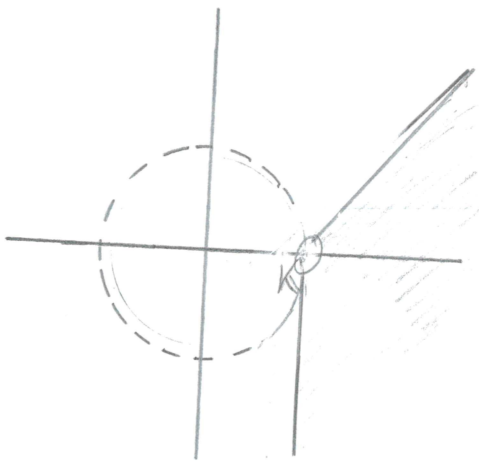
$y = 1$



$|z| = \sqrt{(-1)^2+1^2}$

$= \underline{\sqrt{2}}$

9



Infinetly Many!

w can take any value in the shaded region as this is the region that satisfies both inequalities.

(10) $|z_1 z_2| = |z_1||z_2|$

$$|z_1| = \sqrt{(2p)^2 + (-3p)^2}$$
$$= \sqrt{4p^2 + 9p^2}$$
$$= \sqrt{13p^2}$$
$$= \sqrt{13}p$$

$$|z_2| = \sqrt{(p)^2 + (2\sqrt{3}p)^2}$$
$$= \sqrt{p^2 + 12p^2}$$
$$= \sqrt{13p^2}$$
$$= \sqrt{13}\,p$$

$$\therefore |z_1 z_2| = \sqrt{13}p \times \sqrt{13}p$$
$$= 13p^2$$

$$13p^2 = 52$$
$$p^2 = 4$$
$$p = \pm 2$$
$p = 2$ as $p > 0$