

Algebraic Fractions (Solving Equations)

$$(1) \frac{1}{x} + \frac{1}{x+1} = \frac{7}{12}$$

$$(a) \frac{1(x+1) + 1(x)}{x(x+1)} = \frac{7}{12}$$

$$\frac{x+1+x}{x(x+1)} = \frac{7}{12}$$

$$\frac{2x+1}{x(x+1)} = \frac{7}{12} \checkmark$$

(b) 'Cross multiplying'

$$12(2x+1) = 7x(x+1)$$

$$24x+12 = 7x^2+7x$$

$$0 = 7x^2+7x-24x-12$$

$$0 = 7x^2-17x-12 \checkmark$$

(c) Factorising $7x^2+4x-21x-12=0$

$$0 = 7x(x-3) + 4(x-3)$$

$$0 = (7x+4)(x-3)$$

$$\therefore x = -\frac{4}{7} \text{ or } x = 3$$

Quadratic Equation

$$a=7, b=-17, c=-12$$

$$x = \frac{-(-17) \pm \sqrt{(-17)^2 - 4(7)(-12)}}{2(7)}$$

$$x = 3 \text{ or } x = -\frac{4}{7}$$

$$(2) (a) \frac{1(x+3) + 2(x+1)}{(x+1)(x+3)} = \frac{17}{35}$$

$$\frac{3x+5}{(x+1)(x+3)} = \frac{17}{35}$$

$$35(3x+5) = 17(x+1)(x+3)$$

$$105x+175 = 17x^2+68x+51$$

$$0 = 17x^2+68x-105x+51-175$$

$$0 = 17x^2-37x-124 \checkmark$$

(b) Quadratic equation:

$$a=17, b=-37, c=-124$$

$$x = \frac{-(-37) \pm \sqrt{(-37)^2 - 4(17)(-124)}}{2(17)}$$

$$x = 4 \text{ or } x = -\frac{31}{7}$$

$$(3) (a) \frac{4x-3(x+6)}{x(x+6)} = \frac{5}{2}$$

$$2(x-18) = 5x(x+6)$$

$$0 = 5x^2+28x+36$$

$$x = -2 \text{ or } x = -3.6$$

$$(b) \frac{3(x+2) + 4(x-1)}{(x-1)(x+2)} = \frac{-39}{14}$$

$$14(7x+2) = -39(x-1)(x+2)$$

$$98x+28 = -39x^2-39x+78$$

$$39x^2+137x-50=0$$

$$x = \frac{1}{3} \text{ or } 0.3 \quad x = -286.29 \dots$$

$$\therefore -286 \text{ to } 355$$

$$(3) (c) \frac{x}{4} + \frac{2}{x+7} = \frac{17}{12}$$

$$\frac{x(x+7) + 2(4)}{4(x+7)} = \frac{17}{12}$$

$$\frac{x^2 + 7x + 8}{4(x+7)} = \frac{17}{12}$$

$$12(x^2 + 7x + 8) = 68(x+7)$$

$$12x^2 + 16x - 380 = 0$$

$$x = 5 \text{ or } x = -6.3$$

$$(a) \frac{5}{2x+1} - \frac{3}{x-3} = -\frac{22}{9}$$

$$\text{or } \frac{3}{x-3} - \frac{5}{2x+1} = \frac{22}{9}$$

$$\frac{3(2x+1) - 5(x-3)}{(x-3)(2x+1)} = \frac{22}{9}$$

$$9(x+18) = 22(2x^2 - 5x - 3)$$

$$0 = 44x^2 - 119x - 228$$

$$x = 4 \text{ or } x = -1.30$$

$$(e) \frac{4}{3x-1} - \frac{7}{4x+1} = \frac{4}{3}$$

$$\frac{4(4x+1) - 7(3x-1)}{(3x-1)(4x+1)} = \frac{4}{3}$$

$$3(-5x+11) = 4(12x^2 - x - 1)$$

$$0 = 48x^2 + 11x - 37$$

$$x = -1 \text{ or } x = 0.771$$

$$(f) \frac{1}{x-1} + \frac{1}{x+1} = \frac{1}{x}$$

$$\frac{2x}{x^2-1} = \frac{1}{x}$$

$$2x^2 = x^2 - 1$$

$$x^2 = -1$$

No Solutions (No real solutions that is!)
 $x = \pm i$

$$(4) \frac{20}{s} + \frac{30}{s+5} = 2\frac{5}{6}$$

$$(a) \frac{20(s+5) + 30(s)}{s(s+5)} = \frac{17}{6}$$

$$6[50s + 100] = 17s(s+5)$$

$$300s + 600 = 17s^2 + 85s$$

$$0 = 17s^2 - 215s - 600 \checkmark$$

$$(b) a = 17, b = -215, c = -600$$

$$s = 15 \quad s \neq -\frac{40}{7} \quad \therefore s = 15$$

First part 15 mph, 2nd = 20 mph

$$(5) \frac{500}{x} + \frac{600}{x+5} = 49$$

$$\text{or } \frac{500}{x-5} + \frac{600}{x}$$

$$\frac{500(x+5) + 600(x)}{x(x+5)} = 49$$

$$1100x + 2500 = 49x(x+5)$$

$$0 = 49x^2 - 855x - 2500$$

$$x = 20 \quad x \neq -7.349 \dots$$

$$20 \text{ g/cm}^3 \checkmark$$

$$(6) \quad x + 2\left(\frac{1}{x}\right) = \frac{177}{56}$$

$$\frac{x}{1} + \frac{2}{x} = \frac{177}{56}$$

$$\frac{x(x) + 2(1)}{x} = \frac{177}{56}$$

$$56(x^2 + 2) = 177x$$

$$56x^2 - 177x + 112 = 0$$

$$x = \frac{16}{7} \quad \text{or} \quad x = \frac{7}{8}$$

The number is $\frac{16}{7}$

$$(7) \quad \frac{n}{n+1} + \frac{n+2}{n+3} = \frac{505}{264}$$

$$\frac{n(n+3) + (n+1)(n+2)}{(n+1)(n+3)} = \frac{505}{264}$$

$$264[2n^2 + 6n + 2] = 505[n^2 + 4n + 3]$$

$$23n^2 - 436n - 987 = 0$$

$$n = 21 \quad n \neq -\frac{47}{23}$$

$\therefore \frac{21}{22} + \frac{23}{24}$ are the terms