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A LEVEL MATHS

(8) Composite Functions

WORKING AT D/E

(1) $f(x) = (x+1)^2, x \in R$ and $g(x) = 2x - 1, x \in R$
(a) Find (i) $gf(2)$ (ii) $fg(-4)$ (iii) $f^2(-2)$
(b) Show that $fg(x) \equiv Ax^2$ where A is a constant to be found.
(c) Show that $gf(x) \equiv 2x^2 + 4x + 1$
(d) Hence, solve the equation $gf(x) = 1$

(2) $f:(x) \to \frac{1}{x},\ x > 0$ and $g:(x) \to x^2,\ x \in R$
(a) Find a simplified expression for $ff(x)$
(b) Hence, solve the equation $ff(x) = gf(x)$

(3) $f(x) = 4x + 1$
Show that $f^2(x) \neq [f(x)]^2$

WORKING AT B/C

(1) $f(x) = e^{2x}, x \in R$ and $g(x) = \ln(3x-1), x \in R, x > \frac{1}{3}$
(a) Show that $fg(x)$ can be written in the form $(Ax+B)^2$ where A and B are integers to be stated.
(b) Hence, solve the equation $fg(x) = 25$.
(c) Explain why there is only one solution to $fg(x) = 25$
(d) Find the exact solution to the equation
$$gf(x) = ln8$$

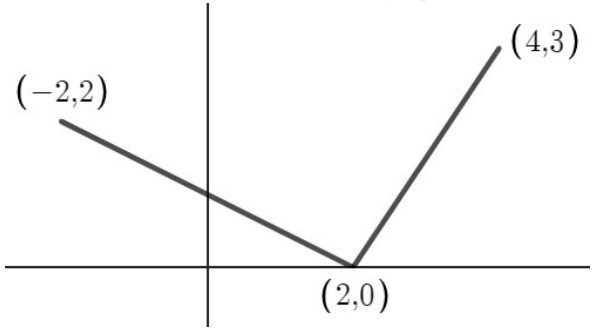
(2) $h(x) = \frac{2}{x-3},\ x \in R,\ x \neq 3$
(a) Show that $h^2(x) = \frac{2x-6}{11-3x}$
(b) Hence, solve the equation $h^2(x) = h(x)$ giving your answers in exact form.

(3) $f(x) = 4 - x,\ x \in R$ and $g(x) = |x|,\ x \in R$
(a) Sketch the graph of $y = gf(x)$
(b) Sketch the graph of $y = fg(x)$

WORKING AT A*/A

(1) $m(x) = 3^x,\ x \in R$ and $n(x) = 2x,\ x \in R$
Solve the equation $mn(x) = n(3) - m(x)$ giving your answer in exact form.

(2) The graph of the linear piecewise function $t(x), -2 \leq t \leq 4$ is shown below. 3 of the points that satisfy $t(x)$ are shown on the graph.



A second function $s(x)$ is such that $s(x) = x^3, x \in R$. Solve the equation $st(x) = 8$

(3) $h(x) = \frac{1}{x}, x \in R, x \neq 0$ and $g(x) = x - 4$
(a) Show that $h(x)$ is self-inverting
(b) Sketch the graphs of $y = hg(x)$ and $y = gh(x)$ on the same set of axes
(c) Hence, solve the equation $hg(x) = gh(x)$ giving your answers in exact form.