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A LEVEL MATHS

(82) Applications of Basic Exponential Models

WORKING AT D/E

(1) Alan is growing a new colony of micro rats in an experiment. The number of rats N after time t weeks from the start of the experiment can be modelled by the equation $N = 10e^{0.2t}$

(a) Write down the initial number of rats at the start of the trial.

(b) Find the number of rats after 20 weeks.

(c) Show that $\frac{dN}{dt} = 2e^{0.2t}$

(d) Find the value of $\frac{dN}{dt}$ when $t = 8$.

(e) Interpret this value in the context of the model.

(f) Sketch the graph of $N = 10e^{0.2t}$ for $t \geq 0$

(g) State a limitation of the model.

WORKING AT B/C

(1) The number of people P after on a newly found island after n years can be modelled by the equation:

$$P = 40e^{0.1n} + 160, \quad n \geq 0$$

(a) Show that there were initially 200 people on the island.

(b) Find the number of people on the island after 12 years.

(c) Show that $\frac{dP}{dn}$ can be written in the form $ke^{0.1n}$ where k is an integer.

(d) What does $\frac{dP}{dn}$ represent in the context of the model?

(e) Find the value of $\frac{dP}{dn}$ when $n = 20$

(f) Sketch the graph of $P = 40e^{0.1n} + 160$

(2) The amount of moss observed on a rock Mkg after time t years can be modelled by the equation

$$M = 2 + 3e^{-\frac{t}{8}}, \; t \geq 0$$

(a) Find the amount of moss initially observed.

(b) Does the equation model growth or decay? You must justify your answer.

(c) Find the amount of moss on the rock after 12 years. Give your answer to the nearest 100g.

(d) Show that $\frac{dM}{dt} = -0.375e^{-\frac{t}{8}}$

(e) Find $\frac{dM}{dt}$ when $t = 9$

(f) Interpret this value in context of the model

(g) Beryl believes there will always be at least 1kg of moss on the rock. Is she correct? You must justify your answer.

(h) Sketch the graph of $M = 2 + 3e^{-\frac{t}{8}}, \; t \geq 0$

WORKING AT A*/A

(1) The value of a boat V £ after t years can be modelled by the equation $V = 8000 + \frac{12000}{e^{\frac{1}{4}t}}, t \geq 0$

(a) Explain why this equation models depreciation.

(b) Find the initial value of the boat.

(c) Find the value of the boat after 8 years giving your answer to the nearest £.

(d) Find the rate at which the boat is depreciating after 10 years.

(e) Sketch the graph of V against t.

(f) Interpret the asymptote on the graph in context of the model.

(g) Make one criticism of the model.

(2) The population of a newly inhabited island can be modelled by the equation $P = 100 + Ae^{bt}$
Where P is the number of people (in thousands) and n is the number of years after the island was first inhabited. A and b are constants.

(a) Given that there were initially 120'000 people on the island, find the value of A.

The rate at which the population is increasing after n years can be found using the expression $6e^{bt}$

(b) Find the value of b.

(c) Find the population after 10 years.

(d) Sketch the graph of P against t.

(e) Use logarithms to find the rate at which the population is increasing at a rate of 40000 people a year.

(3) A model has the equation $A = b + ce^{dt}$ where b, c and d are positive constants and t is time. Find a general expression for the rate at which A is changing.