WWW.M4THS.COM
A LEVEL MATHS

(72) The Applications of Differentiation

WORKING AT D/E

(1) The height of a rocket above the ground (h) in metres after time (t) seconds can be modelled by the equation:

$$h = -t^3 + 2t^2 + 15t, \quad 0 \leq t \leq 3.8$$

(a) Factorise $-t^3 + 2t^2 + 15t$
(b) Hence show that the rocket is only at ground level at the start of the flight.
(c) Find an expression for $h'(t)$
(d) Hence show that the particle is station when $t = 3$
(e) Hence, find the maximum height of the rocket.
(f) Find an expression $h''(t)$
(g) Use your answer to (e) to verify this is a maximum height.
(h) Draw a sketch of $h = -t^3 + 2t^2 + 15t$, $0 \leq t \leq 3.8$

WORKING AT B/C

(1) A piece of wire of length 60cm is bent and made into a rectangle with side lengths x and $2y$.

(a) Show that $2y = 30 - x$

(b) Show that the area (A) of the rectangle can be written as $A = x(30 - x)$

(c) Use differentiation to find the value of x that maximises the area of the rectangle.

(d) Find $\frac{d^2A}{dx^2}$,

(e) Hence, show that this is a maximum value.

(f) Find the maximum area of the rectangle.

(g) Sketch the gradient function $A = x(30 - x)$

Beryl believes there could also be a minimum value for x too.
(h) Explain why she is wrong.

WORKING AT A*/A

(1) The horizontal distance of a car (x) in metres from a fixed point (O) after time (t) seconds can be modelled by the equation

$$x = -t(t - t^{0.5} - 12), \quad 0 \leq t \leq 12$$

(a) State the initial distance of the car from O.

(b) Show that when the car is at its furthest distance from the O, t satisfies the equation:

$$0 = A + Bt^{0.5} + Ct$$

Where A, B and C are integers to be found.

(c) Find the maximum distance from O that the car reaches. Give your answer to 3 SF.

(d) Show that the car never returns to O.