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A LEVEL MATHS

(70) The Applications of Differential Equations

WORKING AT D/E

(1) The rate at which the number of rats in a colony is increasing can be modelled by the differential equation $\frac{dN}{dt} = 0.1N$ where N is the number of rats (in thousands) and t is the time in days.

(a) Show that the general solution to the differential equation is $N = Ae^{0.1t}$ where A is a constant.

(b) Given that there were initially 6000 rats, write down the value of A.

(c) Find the population of the rats after a week.

(d) Sketch the graph of the population of the rats over time.

(e) State a limitation of the model.

WORKING AT B/C

(1) The rate at which a mould patch is shrinking in a shower is proportional the amount already present.

(a) Show that the amount of mould in the shower satisfies the equation $M = Ae^{kt}$ where M is the mass of the mould, t is the time in days and A and k are constants.

(b) Given that there was initially $40g$ of mould and that after 20 days there was $8g$ of mould, find the value of A and k. Give the value of k to 3 significant figures.

(c) Find when the mass of the mould will fall below $2g$.

(d) Sketch the graph of the mass of the mould over time.

WORKING AT A*/A

(1) Juice is being poured into a cooler at a steady rate of $20cm^3s^{-1}$. The cooler is leaking at a rate of one tenth of the current volume (V) of juice in the cooler .

(a) Show that V satisfies the equation

$$V = 200 + Ae^{-0.1t}$$

Where A is a constant and t is the time in seconds.

(b) Given that the initial volume in the cooler is $280cm^3$, find the time taken for volume to reach $250cm^3$

(c) Doris needs to ensure she always has enough in the cooler for one glass of juice. Given the glass holds $180ml$, explain why Doris will always have enough juice in the container for at least one glass.