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A LEVEL MATHS

# (70) Differentiation (Stationary Points)

## WORKING AT D/E

(1) $f(x) = 2x^3 + 4x^2$
(a) Find $f'(x)$
(b) Hence, show that the $x$ coordinates of the two stationary points are $x = 0$ and $x = -\frac{4}{3}$
(c) Hence, find the coordinates of the two stationary points.
(d) Find an expression for $f''(x)$
(e) Find $f''(0)$ and $f''(-\frac{4}{3})$
(f) Hence determine the nature of each stationary point.

(2) $y = 4x^5$
(a) Find an expression for $\frac{dy}{dx}$
(b) Hence find the one stationary point on the curve.
(c) By considering the value of $\frac{dy}{dx}$ when $x = -0.01$ and when $x = 0.01$, explain why the stationary point is a point of inflexion.

(3) $y = \frac{4}{3}x^{\frac{3}{2}} - 18x$
(a) Use differentiation to show that the stationary point on the curve has coordinates $(81, -486)$.
(b) Determine the nature of this stationary point.

## WORKING AT B/C

(1) $f(x) = (x+1)(x-3)(x+2)$
(a) Find an expression for $f(x)$ in the form $f(x) = Ax^3 + Bx^2 + Cx + D$
(b) Use differentiation to show that the $x$ coordinates of the two stationary points on the curve with equation $y = f(x)$ are $x = \pm\frac{\sqrt{21}}{3}$
(c) Find the $y$ coordinate of each stationary point giving each answer to 3SF.
(d) Determine the nature of each stationary point.
(e) Hence, sketch the curve of $y = f(x)$ labelling each stationary point and the points where the curve crosses the coordinate axes.

(2) A curve has equation $y = (x-2)(x^2 + 5x + 10)$
(a) Show that the only root of the equation is $x = 2$
(b) Find any stationary points on the curve.
(c) Find an expression for $\frac{d^2y}{dx^2}$
(d) Using your answer to part (c) show that one of the stationary points is a maximum and one is a minimum.
(e) Hence, sketch the curve of $y = (x-2)(x^2 + 5x + 10)$ labelling each stationary point and the points where the curve crosses the coordinate axes.

## WORKING AT A*/A

(1) A curve has equation $y = \frac{x^{\frac{4}{3}} - x}{\sqrt{x}}$, $x > 0$
(a) Find an expression for $\frac{dy}{dx}$ in the form $Ax^n(B + Cx^m)$
(b) Hence, show that the $x$ coordinate of the stationary point is $x = \frac{27}{125}$
(c) Prove that this is a minimum point.

(2) Determine the least value of the function $g(x) = 2x^4 + 64x$

(3) Prove that $f(x) = x^3 - 3x^2 + 18x + 12$ is an increasing function for all values of $x$.