

(69) Solving Differential Equations

WORKING AT D/E

(1) Find a general solution to each differential equation:

(a) $\frac{dy}{dx} = \frac{x}{e^y}$ (b) $\frac{dy}{dx} = \frac{\cos 2x}{\sin y}$ (c) $\frac{dy}{dx} = y \sec^2 x$

(2) Give that when $x = 1, y = \frac{\pi}{2}$ show that the particular solution to the differential equation

$$x \frac{dy}{dx} = \sec y \text{ can be written as } x = e^{\sin(y)-1}$$

(3) Find the particular solution to the differential equation $\frac{dy}{dx} = y^2 e^{2x}$ given the equation satisfies the boundary conditions $y = 1$ at $x = 0$. Give your answer in the form $y = f(x)$

WORKING AT B/C

- (1) (a) Express $\frac{14-2x}{(1+x)(3-x)}$ in partial fractions.
(b) Hence, find a general solution to the differential equation $(1+x)(3-x) \frac{dy}{dx} = \frac{2(7-x)}{y}$ in the form $y^2 = f(x)$

(2) The differential equation $\cos^2(x) \frac{dy}{dx} = y$ has boundary condition $y = 1$ at $x = \frac{\pi}{4}$.

Show that the origin $(0, \frac{1}{e})$ also satisfies the equation.

(3) A differential equation is such that $\frac{dy}{dx} = x^2$

On the same set of axes, draw 3 different particular solutions to the differential equation.

WORKING AT A*/A

(1) Show that the particular solution to the differential equation $e^{y-x} \frac{dy}{dx} = -x$ with boundary conditions $y = 0$ at $x = 0$ can be written as $y = x + \ln(1-x)$

(2) Find a general solution to the differential equation $\frac{dy}{dx} = (\ln x) \operatorname{cosec}^2(y)$

(3) Given that one particular solution to the differential equation $\frac{dy}{dx} = 3^x e^{-3y}$ passes through the origin, prove that $\ln 3(e^{3y} - 1) + 3 = 3^{x+1}$