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A LEVEL MATHS

(66) Integration using Partial Fractions

WORKING AT D/E

(1) (a) Using partial fractions, show that $\frac{5+4x}{(1+x)(2+x)}$ can be written as $\frac{1}{1+x}+\frac{3}{2+x}$

(b) Hence, show that $\int \frac{5+4x}{(1+x)(2+x)}dx$ can be written as the sum of natural logarithms.

(2) (a) Express $\frac{x+13}{(1-x)(6+x)}$ in the form $\frac{A}{(1-x)}+\frac{B}{(6+x)}$

(b) Hence, using integration and the laws of logarithms, show that

$$\int \frac{x+13}{(1-x)(6+x)}dx = \ln\frac{|6+x|}{(1-x)^2}+c$$

WORKING AT B/C

(1) Doris wants to express $\frac{-1}{(1+2x)(1+x)^2}$ in partial fractions. She writes

$$\frac{-1}{(1+2x)(1+x)^2} \equiv \frac{A}{(1+2x)}+\frac{B}{(1+x)^2}$$

(a) Explain the error that she has made.

(b) Show, using partial fractions that

$$\frac{-1}{(1+2x)(1+x)^2} \equiv \frac{2}{(1+x)}+\frac{1}{(1+x)^2}-\frac{4}{(1+2x)}$$

(c) Hence, show that

$$\int \frac{-1}{(1+2x)(1+x)^2}dx = \ln\left(\frac{1+x}{1+2x}\right)^2-\frac{1}{1+x}+c$$

(2) (a) Factorise $1-16x^2$

(b) Hence, express $\frac{5+4x}{1-16x^2}$ in partial fractions

(c) Show that $\int_0^{0.1}\left(\frac{5+4x}{1-16x^2}\right)dx = \ln\left(\frac{\sqrt{1.4}}{\sqrt[4]{0.216}}\right)$

WORKING AT A*/A

(1) (a) Show that $\frac{11+4x-2x^2}{6+x-x^2}$ can be written in the form

$$A+\frac{B}{3-x}-\frac{C}{2+x}$$

(b) Hence, show that

$\int_0^2 \frac{11+4x-2x^2}{6+x-x^2}dx = p+\ln q$ where p and q are rational constants to be found.