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A LEVEL MATHS

(64) Integration by Substitution

WORKING AT D/E

(1) (a) Given that $u = 3x + 2$, find an expression for $\frac{du}{dx}$ in terms of x.
(b) Using your answer to part (a) show that $\int e^{3x+2}\, dx$ can be written as $\int \frac{1}{3} e^u\, du$
(c) Find $\int \frac{1}{3} e^u\, du$ in terms of u.
(d) Hence, find $\int e^{3x+2}\, dx$ in terms of x

(2) (a) Using the substitution $u = \sin x$ show that $\int 2\cos x\, e^{sinx}\, dx$ can be written as $\int 2e^u\, du$

(b) Use your answer to part (a) to find $\int 2\cos x\, e^{sinx}\, dx$ in terms of x

(3) (a) Using the substitution $u = x - 6$, show that $\int x\sqrt{x-6}\ dx$ can be written as $\int (u+6)u^{\frac{1}{2}}\, du$

(b) Hence, show that

$$\int x\sqrt{x-6}\ dx = \frac{2}{5}(x-6)^{\frac{5}{2}} + 4(x-6)^{\frac{3}{2}} + c$$

WORKING AT B/C

(1) (a) Using the substitution $u = \sin x$, show that $\int \cos x \sin^7 x\ dx = \frac{1}{8}\sin^8 x + c$

(b) Using the substitution $u = 4x - 3$ show that $\int 16x\sqrt[3]{4x-3}\, dx = \frac{3}{7}(4x-3)^{\frac{7}{3}} + \frac{9}{4}(4x-3)^{\frac{4}{3}} + c$

(2) Using thew substation $u = \tan x$, show that

$$\int_0^{\frac{\pi}{4}} \sec^2 x e^{\tan x}\, dx = e - 1$$

(3) Using the substitution $u = 1 + \sin 2x$, show that

$$\int_0^{\frac{\pi}{4}} \frac{\cos 2x}{1+\sin} dx = \ln\sqrt{2}$$

WORKING AT A*/A

(1) Prove that $\int \frac{1}{(1-x^2)^{\frac{1}{2}}} dx = \arcsin x + c$

(2) Use the substitution $u^3 = 6x + 1$ to show that $\int \frac{3x}{\sqrt[3]{6x+1}} dx = \frac{3}{40}(4x-1)(6x+1)^{\frac{2}{3}} + c$

(3) Use the substitution $u^2 = e^x + 1$ to find $\int_0^{\ln 3} \frac{e^{3x}}{e^x+1}\, dx$ giving your answer in the form $a + \ln b$