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A LEVEL MATHS

(62) Integrating using Trigonometric Identities

WORKING AT D/E

(1) (a) Write $\tan^2 x$ in terms of $\sec x$

(b) Hence, using the formula book, show that $\int_0^{\frac{\pi}{4}} \tan^2 x \, dx = 1 - \frac{\pi}{4}$

(2) (a) Using the formula book, show that $\cos(2x)$ can be written as $\cos^2 x - \sin^2 x$

(b) Hence, show that $\cos(2x) = 1 - 2\sin^2 x$

(c) Using your answer to part (b), show that $\int \sin^2 x \, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$

(3) (a) Using the formula book, write $\sin 2x$ in terms of $\cos x$ and $\sin x$

(b) Hence, find $\int \sin x \cos x \, dx$

WORKING AT B/C

(1) (a) Prove that $(\cos x + \sin x)^2 \equiv 1 + \sin 2x$

(b) Hence, find $\int_0^{\frac{\pi}{6}} (\cos x + \sin x)^2$ giving your answer in exact form.

(2) (a) Write $\cot^2 x$ in terms of $\operatorname{cosec}^2 x$

(b) Hence, find $\int 3\cot^2 x \, dx$

(3) (a) Show that $(\cos x + \tan x)^2$ can be written as:

(i) $\cos^2 x + 2\sin x + \tan^2 x$

(ii) $\frac{1}{2}\cos 2x + 2\sin x + \sec^2 x - \frac{1}{2}$

(b) Using your answer to part (ii), find

$$\int (\cos x + \tan x)^2 \, dx$$

WORKING AT A*/A

(1) Show that:

$$\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} (1 - \sin 2x)^2 = \frac{1}{16}\left(3\pi + 2 - 8\sqrt{2}\right)$$

(2) Evaluate $\int_0^{\pi} \left(4\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) + \left(2\cos^2\frac{\theta}{2} - 1\right) \, d\theta$

(3) (a) Show that $\frac{1 + \cos x - \cos^2 x}{\sin x} \equiv \cot x + \sin x$

(b) Hence, find $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1 + \cos x - \cos^2 x}{\sin x} \, dx$ giving your answer in the form $a + \ln b$