

(61) Integrating Functions of the form $f(ax + b)$

WORKING AT D/E

(1) Find each of the following by using the formula book:

(a) $\int -2 \sec 2x \tan 2x \, dx$ (b) $\int \tan 3x \, dx$
 (c) $\int -4 \operatorname{cosec}^2 2x \, dx$ (c) $\int 8 \operatorname{cosec} 4x \cot 4x \, dx$

(2) Show each of the following results. You must show your full method:

(a) $\int_0^1 e^{2x} \, dx = \frac{1}{2}(e^2 - 1)$

(b) $\int_0^{\frac{\pi}{2}} \cos\left(2x - \frac{\pi}{6}\right) \, dx = \frac{1}{2}$

(c) $\int_0^2 \frac{2x}{x^2+3} \, dx = \ln \frac{5}{3}$

(3) Show each of the following results:

(a) $\int_0^1 (3x + 1)^2 \, dx = 7$

(b) $\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} -2 \operatorname{cosec} 2x \cot 2x \, dx = \frac{2\sqrt{3}}{3} - 2$

WORKING AT B/C

(1) Find $\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$ showing full workings.

(2) Find each of the following integrals:

(a) $\int \frac{4x+2}{2x^2+2x} \, dx$

(b) $\int e^{3x} + \operatorname{cosec}^2 4x \, dx$

(c) $\int e^{2x+3} - \frac{3}{x} \, dx$

(d) $\int 4x + \sin(2-x) \, dx$

(3) Evaluate each of the following. You must show full workings and give answers in exact form:

(a) $\int_1^2 \frac{2x+3}{x^2+3x} \, dx$

(b) $\int_0^{\frac{\pi}{8}} -2 \sin 4x \, dx$

(c) $\int_0^2 \frac{1}{3} e^{4x+1} \, dx$

WORKING AT A*/A

(1) (a) Show that

$$(\operatorname{cosec} x + \tan x)^2 \equiv \operatorname{cosec}^2 x + 2 \sec x + \sec^2 x - 1$$

(b) Hence find $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\operatorname{cosec} x + \tan x)^2 \, dx$ giving your answer in exact form.

(2) Evaluate $\int_1^3 \frac{2x+7}{2x^2+14x} \, dx$ giving your answer in the form $\ln \frac{\sqrt{a}}{b}$ where \sqrt{a} and b are in their simplest form.

(3) Given that $\int_1^{e^4} \frac{p}{x} \, dx = \frac{16}{3}$, where p is a positive constant, find the value of p .