

## (60) Integrating Standard Functions (Logs and Trig)

### WORKING AT D/E

(1) Use the formula book to find the following integrals:

(a)  $\int \sec^2 x \, dx$                       (b)  $\int \sec x \tan x \, dx$   
 (c)  $\int \operatorname{cosec}^2 x \, dx$                 (d)  $\int -\operatorname{cosec} x \cot x \, dx$

(2) Considering the derivatives of  $\sin x$ ,  $\cos x$ ,  $\ln x$  and  $e^x$  find the following integrals:

(a)  $\int \cos x \, dx$                       (b)  $\int \sin x \, dx$   
 (c)  $\int \frac{2}{x} \, dx$                             (d)  $\int e^{3x} \, dx$

(3) Using the formula book and the results above, find each integral below:

(a)  $\int \tan x - \frac{1}{x} + e^x \, dx$   
 (b)  $\int -2 \operatorname{cosec}^2 x - \sin x \, dx$   
 (c)  $\int 4 \sec^2 x - e^x - x \, dx$   
 (d)  $\int \cot 2x - x^{-2} - \cos x \, dx$

### WORKING AT B/C

(1) (a) Show that  $\frac{4-x^2}{x} \equiv \frac{A}{x} + Bx$   
 (b) Hence, find  $\int \frac{4-x^2}{x} \, dx$   
 (c) Simplify  $\sin x (1 + \cot x)$   
 (d) Hence, find  $\int \sin x (1 + \cot x) \, dx$

(2) (a) Find  $\int \frac{5}{x} - \frac{1}{\sin^2 x} \, dx$

(b) (i) Show that  $\frac{\sin x}{\cos^2 x} \equiv \sec x \tan x$

(ii) Hence, find  $\int \frac{\sin x}{\cos^2 x} \, dx$

(3) Find the value of each giving your answers in exact form where appropriate. You must show full workings :

(a)  $\int_0^{\frac{\pi}{4}} x - \sec^2 x \, dx$

(b)  $\int_1^{3\frac{4}{x}} - e^x + x \, dx$

(c)  $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin x - \operatorname{cosec}^2 x \, dx$

(d)  $\int_1^6 \frac{8-3x}{x} \, dx$

### WORKING AT A\*/A

(1) Evaluate  $\int_0^{\frac{\pi}{4}} \sec \theta (\sec \theta - \sin \theta) \, d\theta$  giving your answer in the form  $\ln Ae$  where  $A$  is a simplified surd.

(2) Show that  $\int_1^2 \frac{(1-x)^2}{2x} \, dx = A \ln B + \frac{C}{D}$  where  $A, B, C$  and  $D$  are rational constants to be found.

(3) Show that

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \operatorname{cosec} x (\cos x + \sin x)^2 \, dx = 2 - \sqrt{3} + \ln \sqrt{3}$$