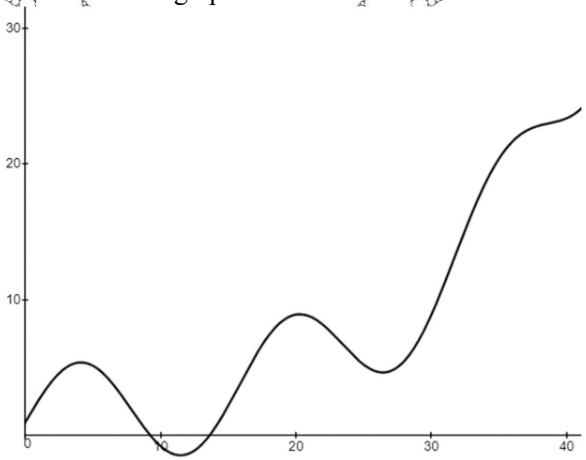
WWW.M4THS.COM
A LEVEL MATHS

(59) Applications of Numerical Methods

WORKING AT D/E

(1) The flight of a bird is modelled by the equation $y = 4\sin(0.4t) + e^{0.08t}$ where y is the height in metres above sea leave of the bird at time t seconds after it takes off.
Approximately 40 seconds of the bird's flight is modelled in the graph below.

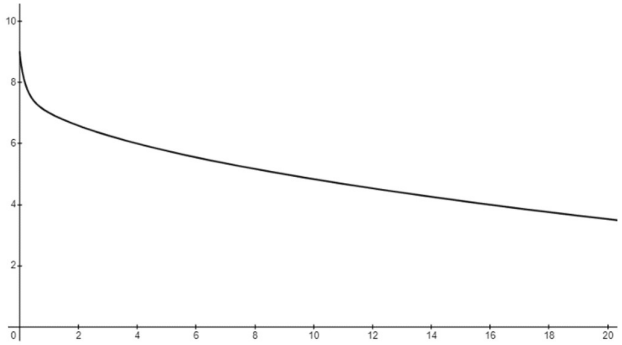


(a) Using the graph, estimate when the bird was first below sea level
(b) Prove this time was in the interval $9.2 < t < 9.3$
(c) Show that the bird returns to a height above sea level in the interval $13.6 < t < 13.7$
(d) Hence, **estimate** the length of time the bird was above sea leave in the first 40 seconds of its flight.

WORKING AT B/C

(1) The amount of a drug in a patient's body after it is first administered can be modelled by the equation $N = 8 - \sqrt{t} + e^{-0.01t}$ where N is the amount of the drug (in mg) and t is the number of hours after the drug is first given.
(a) Show that the initial dose of the drug was 9mg.

The diagram below shows the graph of the model for the first hours.



Doris wants to find out an approximation of how long it took before only 5mg of the drug remained in the patient's body.
(b) Show that Doris can use the equation $0 = -\sqrt{t} + e^{-0.01t} + 3$ to find an approximation for this time.
(c) Show that the time taken for there to be 5mg was between 14 hours and 54 minutes and 15 hours.
(d) Taking $t_0 = 15$ use the iterative formula $t_{n+1} = (e^{-0.01t_n} + 3)^2$ find t_1, t_2 and t_3 to 4 dp.
(e) Hence explain why the time taken was closer to 14 hours and 54 minutes than 15 hours.

WORKING AT A*/A

(1) A girl jumps from a diving board into a swimming pool. The height (H) of a girl above a swimming pool (t) seconds after she jumps can be modelled by the equation $H = 10 - t - 2^t$ where H is measured in metres.
(a) State the height above the water from which she jumps.
(b) Show that the girl hits the water between 2.5 and 3 seconds.
(c) Using 3.1 seconds as an initial approximation for the time the girl hits the water, use the Newton-Raphson method to find 4 further approximations for the time it takes to hit the water giving each approximation to 4 dp.
(d) Sketch the graph of H against t
(e) State one limitation of the model.