WWW.M4THS.COM
A LEVEL MATHS

(58) Numerical Methods Newton-Raphson Method

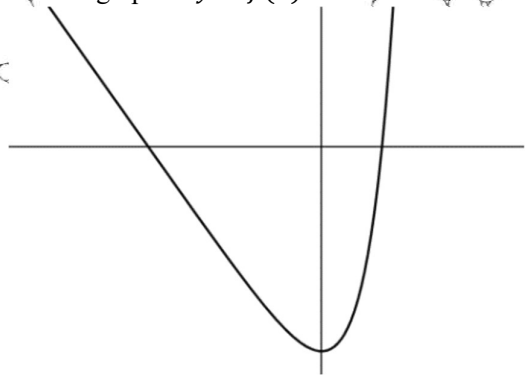
WORKING AT D/E

(1) $f(x) = e^x - x - 6, \quad x \epsilon R$

A root to the equation is α

(a) Show that $-6.0 < \alpha < -5.9$

Part of the graph of $y = f(x)$ is shown below



(b) Mark α on the diagram

(c) Find an expression for $f'(x)$

(d) **The Newton-Raphson formula is given in the formula book.** Using $x_0 = -5.5$ as an initial approximation for α, use the Newton-Raphson method to find x_1, x_2, x_3 and x_4, giving each approximation to 6 dp.

(e) Show that $\alpha = -5.9975$ correct to 4 decimal places.

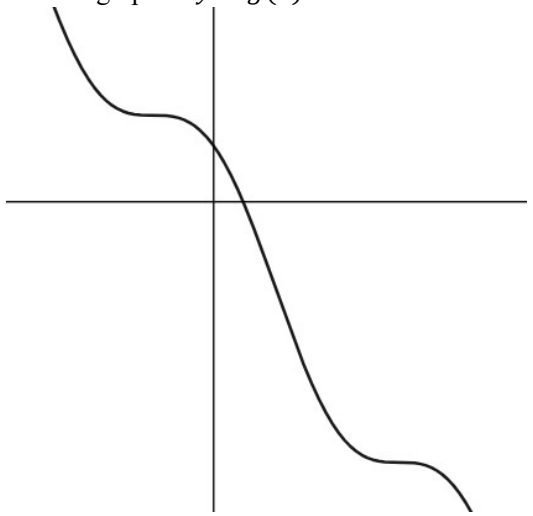
(f) β is the only other root of $f(x)$. Explain why $\beta > 0$.

(g) Mark on the diagram where $f'(x) = 0$

WORKING AT B/C

(1) $g(x) = \cos(x) - x, \quad x \epsilon R$ where x is measured in radians.

Part of the graph of $y = g(x)$ is shown below



(a) Mark any point on the diagram where it would not be appropriate to use the Newton-Raphson method to locate a root to the equation.

(b) Show that a root, α, of the equation $g(x) = 0$ is such that $0.7 < \alpha < 0.8$

(c) Using $x_0 = 0.85$ as an initial approximation for β, use the Newton-Raphson method to find x_1, x_2, x_3 and x_4, giving each approximation to 4 dp.

(d) Show that $\alpha = 0.739$ to 3 SF.

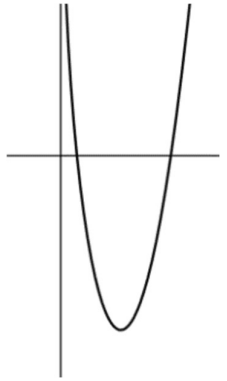
(e) By considering the range of $\cos(x)$ explain why there are no more roots of $g(x)$

(f) Explain why there are more stationary points on the curve $y = g(x)$

WORKING AT A*/A

(1) $f(x) = 3x^2 - 4x - \ln 2x, \quad x > 0$

Part of the curve of $y = f(x)$ is shown in the diagram below.



The two roots to the equation are α and β where $\beta > \alpha$.

(a) Show that $0.2 < \alpha < 0.3$

(b) Show that $1.5 < \beta < 1.6$

(c) The curve has a stationary point in the interval $\gamma < x < \delta$. Write down possible values of γ and δ.

(d) Show that $f(x)$ is stationary in the interval $0.8 < x < 0.9$

(e) Cyril wants to find the root β to 3 decimal places. He decides to use the Newton-Raphson method to locate the root. He takes $x_0 = 0.9$. Comment on his approach.

(f) Using $x_0 = 1.3$ as an initial approximation for β, use the Newton-Raphson method to find x_1, x_2, x_3 and x_4, giving each approximation to 4 dp.

(g) State a suitable starting value for x_0 to find an approximation for finding α using the Newton-Raphson method,

(h) Explain what the formula $x_{n+1} = x_n - \frac{6x - 4 - x_n^{-1}}{6 + x_n^{-2}}$ could be used for.