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A LEVEL MATHS

(57) Numerical Methods Iteration to Locate Roots

WORKING AT D/E

(1) $f(x) = x^2 - 4x + 1,\ x \epsilon R$

(a) Show that there is a root α to $f(x)$ in the interval $[3.7, 3.8]$

(b) Using the iterative formula $x_{n+1} = \sqrt{4x_n - 1}$, $x_0 = 4$, to find the value of x_1, x_2 and x_3 giving your answers to 5 decimal places.

(c) State the suitability of the iterative formula for locating α.

(d) Prove that $\alpha = 3.732$ correct to 3 decimal places.

(e) Doris decides to use the use the iterative formula $x_{n+1} = \frac{{x_n}^2+1}{4}$ to locate the other root of the $f(x)$. Taking $x_0 = 1$, show that the other root $\beta \approx 0.2679$

(2) $f(x) = x^5 + 5x^2 - 3,\ x \epsilon R$

(a) Show that there is a root α to the equation $x^5 + 5x^2 - 3 = 0$ in the interval $0.7 < x < 0.8$

(b) Show that $f(x) = 0$ can be written as $x = \sqrt{\frac{3-x^5}{5}}$

(c) Taking $x_0 = 0.5$, use the iterative formula $x_{n+1} = \sqrt{\frac{3-{x_n}^5}{5}}$ to find the values of x_1, x_2 and x_3 giving your answers to 5 decimal places.

(d) Prove that $\alpha = 0.744$ correct to 3dp.

WORKING AT B/C

(1) $f(x) = x - \sin 2x,\ x \in R$

(a) Show that there is a root α to $f(x)$ in the interval $[0.9, 1.0]$

$f(x) = 0$ can be written as either:

(i) $x = \frac{\sin^{-1} x}{2}$ (ii) $x = \sin 2x$

(b) Explain why using the iterative formula $x_{n+1} = \frac{\sin^{-1} x_n}{2}$, with $x_0 = 0.8$ doesn't locate α.

(c) Using the iterative formula $x_{n+1} = \sin 2x_n$ with $x_0 = 0.8$, find the values of x_1, x_2 and x_3 giving your answers to 5 decimal places.

(d) Prove that $\alpha = 0.9477$ correct to 4 S.F.

(2) $f(x) = 2x^2 - 2x + e^{5x},\ x \epsilon R$

(a) Find $f'(x)$

$f(x)$ has a stationary point β.

(b) Use your answer to part (a) to show that the x coordinate of β is in the interval $(-0.2, -0.1)$

(c) Use the iterative formula $x_{n+1} = 0.2 \ln\left(\frac{2-4x_n}{5}\right)$ with $x_0 = -0.1$ to find the values of x_1, x_2, x_3, x_4, x_5 and x_6 giving your answers to 5 decimal places.

(d) Prove that the stationary point on the curve with equation $y = f(x)$ has x coordinate -0.135 correct to 3 S.F

(e) Explain why the iterative formula $x_{n+1} = \frac{2-5e^{5x_n}}{4}, x_0 = -0.1$ doesn't locate the x coordinate of β

WORKING AT A*/A

(1) (a) On the same set of axes, sketch the graphs of $y = 1 - \ln(x+1),\ x > -1$ and $y = x,\ x \in R$

(b) Explain why there is one root to the equation $x = 1 - \ln(x+1)$

The function $f(x) = \ln(x+1) + x - 1,\ x > -1$

(c) Show that there is a root α to $f(x)$ in the interval $[0.5, 0.6]$

(d) Using the iterative formula $x_{n+1} = 1 - \ln(x_n + 1),\ x_0 = 0.5$ find the values of x_1, x_2, x_3, x_4, x_5 and x_6 giving your answers to 5 decimal places.

(e) Using your answer to part (d) explain why the iterations found in part (c) create a cobweb diagram.

(2) (a) On the same set of axes, sketch the graphs of $y = e^{4x} - 3$ and $y = x$

$f(x) = e^{4x} - 3 - x,\ x \in R$

(b) Using your answer to part (a), explain why there is a root to the equation $f(x)$ for $x > 0$

(c) Show that the root $0.2 < \alpha < 0.3$

(d) Show that the equation $f(x) = 0$ can be written as $e^{4x} - 3 = x$

(e) Doris uses the iterative formula $x_{n+1} = e^{4x_n} - 3, x_0 = 0.3$ to try and locate α. Find the values of x_1, x_2, x_3 and x_4 giving your answers to 5 decimal places.

(f) With the aid of a diagram, comment on the likely success of Doris' attempt.