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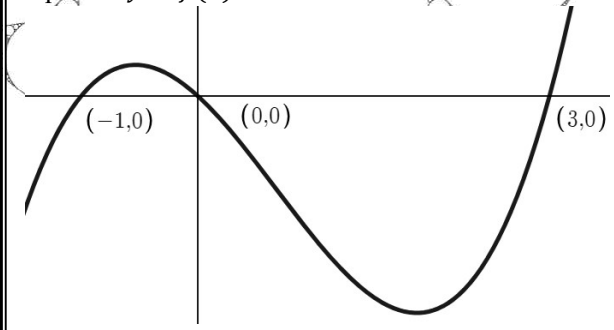
A LEVEL MATHS

(56) Numerical Methods Locating Roots

WORKING AT D/E

(1) $f(x) = x(x+1)(x-3),\ x \in R$
The diagram below shows part of the curve with equation $y = f(x)$



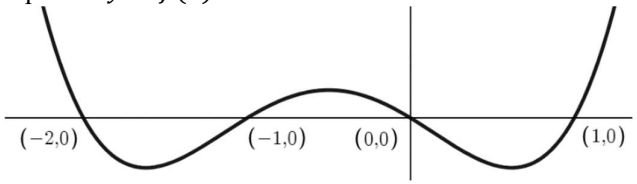
(a) Write down the roots of the equation.
(b) Find the value of (i) $f(2.9)$ (ii) $f(3.1)$
(c) Explain how your answers in part (b) confirm that there is root in the interval (2.9, 3.1)

(2) $g(x) = \ln(x-5),\ x > 5$
(a) Find the value of (i) $f(5.9)$ (ii) $f(6.1)$
(b) What does your answer to part (b) tell you?

(3) Doris wants to locate a root of the equation $h(x) = \frac{1}{x}$. She finds the value of $h(-0.1)$ and $h(0.1)$. Comment on her method.

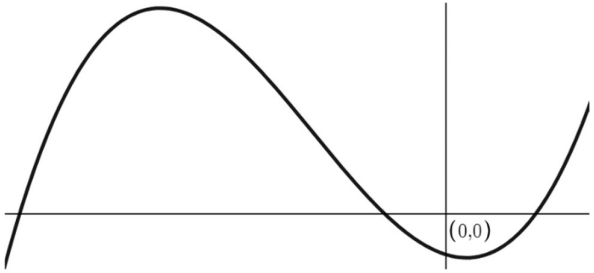
WORKING AT B/C

(1) The diagram below shows part of the curve with equation $y = f(x)$



Cyril finds the value of $f(-1.1)$ and $f(0.1)$. He deduces that there is not root in the interval $-1.1 < x < 0.1$
(a) Why do you think he made this deduction?
(b) Comment on his findings.

(2) The diagram below shows part of the curve with equation $y = x^3 + 3x^2 - x - 1$



Show that there is a root of the equation in the interval [0.5,0.75]

(3) (a) Sketch the graphs of $y = x$ and $y = \cos x, -2\pi \le x \le 2\pi$ on the same set of axes.

(b) Using your sketch, write down the number of roots to the equation $x = \cos x$

(c) Show that there is a root is in the interval (0.7,0.8)

WORKING AT A*/A

(1) $f(x) = e^{2x} - \ln x, x > 0$

(a) Show there is a stationary point in the interval [0.28,0.29]

(b) Hence, determine the nature of the stationary point.

(c) The x coordinate of the stationary point is α. Show that $\alpha = 0.284$ correct to 3 decimal places.

(2) (a) On the same set of axes, sketch the graphs of $y = \sin x$ and $y = e^{-x}, 0 < x < \pi$

(b) Using your answer to part (a) state the number of roots to the equation $e^x = \operatorname{cosec} x$

(c) Show that a root to the equation $e^x = \operatorname{cosec} x$ lies in the interval $0.5 < x < 0.6$

(3) $f(x) = e^x + \tan x,\ \frac{-\pi}{2} < x < \frac{\pi}{2}$.

(a) $f(x)$ has a root α. Show that $-0.6 < \alpha < -0.5$

(b) Cyril wants to find more roots to the equation outside the original domain. He finds that $f(1.5)$ is positive and $f(1.6)$ is negative. He says there must be a root in this interval. Explain he wrong.