

## (53) Proving Trigonometric Identities

### WORKING AT D/E

(1) Write down the two trigonometric identities that you will need to use in this unit.

(2) Simplify each of the following using your answers from question (1) to help you.

(a)  $\frac{\sin(6x)}{\cos(6x)}$

(b)  $\sqrt{1 - \cos^2(x)}$

(c)  $\sqrt{1 - \sin^2(3x)}$

(d)  $1 - \sin^2(x)$

(e)  $\sin^2(8x) + \cos^2(8x)$

(f)  $6\sin^2(\theta) + 6\cos^2(\theta)$

(g)  $\frac{\sin^2(x)}{\cos^2(x)}$

(h)  $\frac{\sqrt{1 - \cos^2(4\theta)}}{\sqrt{1 - \sin^2(4\theta)}}$

(i)  $\tan(x) \cos(x)$

(3) Show that

$$(\sin(x) + \cos(x))^2 \equiv 2 \sin(x) \cos(x) + 1$$

### WORKING AT B/C

(1) Using the identity

$$a^4 - b^4 \equiv (a^2 - b^2)(a^2 + b^2)$$

Show that

$$\cos^4 x - \sin^4 x \equiv \cos^2 x - \sin^2 x$$

(2) Prove each identity:

(a)  $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \equiv \frac{1}{\sin \theta \cos \theta}$

(b)  $\frac{3 \sin(A)}{\tan(A)} \equiv 3 \cos(A)$

(3) (a) Given that  $9 \sin x = 14 \cos x$ , write down the value of  $\tan x$

(b) Given that  $0 < A < 90$  and  $\sin(A) = \frac{3}{5}$

(i) Show that  $\cos(A) = \frac{4}{5}$

(ii) Find the value of  $\tan(A)$

### WORKING AT A\*/A

(1) (a) Given that  $180 < A < 270$  and  $\sin(A) = -0.8$

(i) Find the exact value of  $\cos(A)$

(ii) Find the exact value of  $\tan(A)$

(b) How would your answer(s) change if  $270 < A < 360$ ?

(2) (a) Given that  $x = 4 \cos \theta$  and  $y = 2 + 4 \sin \theta$ , show that  $x^2 + (y - 2)^2 = k$  where  $k$  is a constant to be found.

(b) Given that  $p = 1 - 2 \cos x$  and  $q = 3 \sin x + 1$  show that  $9p^2 + 4q^2 - 18p - 8q - 23 = 0$

(3) Prove each identity

(a)  $\sin(90 - x) \tan(x) \equiv \sin(x)$

(b)  $\frac{(\sin(x) + \cos(x))^2}{\sin(x) \cos(x)} \equiv 2 + \frac{1}{\sin(x) \cos(x)}$

(c)  $\tan A + \sin A \equiv \frac{\sin A(1 + \cos A)}{\cos A}$

(d)  $\frac{\cos^4 x - \sin^4 x}{\cos^2 x - \sin^2 x} \equiv 1$

(e)  $\sin x \sqrt{1 + \tan^2 x} \equiv \tan x$