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A LEVEL MATHS

(52) Differentiating Parametric Equations

WORKING AT D/E

(1) Find an expression for $\frac{dy}{dx}$ in terms of the parameter t for each of the following pairs of parametric equations:

(a) $x = 3t,\ y = t^2$ (b) $x = e^{2t}, y = \cos t$

(c) $x = \sqrt{t}, y = 4t^3$ (d) $x = \cos t, y = \sin t$

(2) A curve has parametric equations
$x = -\cos t,\ \ y = \sin t,\ \ 0 < t < \frac{\pi}{2}$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(b) Hence, show that the equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$ is $y = x - \sqrt{2}$

(3) A curve has parametric equations
$x = \ln t,\ y = t^2,\ t > 0$

(a) Show that $\frac{dy}{dx} = 2t^2$

(b) Hence, find the equation of the tangent at the point where $t = 1$

WORKING AT B/C

(1) A curve has parametric equations
$x = 8 - t^2,\ y = t^3,\ t \epsilon R$

Find the equation of the normal to the curve at the point where $t = 4$ in the form $ax + by + c = 0$

(2) The curve C has parametric equations
$x = 2\sin t - t,\ \ y = \cos t + 3,\ \ 0 < t < \frac{\pi}{2}$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(b) Hence, find the coordinates of the stationary point on the curve.

(3) A curve has parametric equations
$x = \ln t,\ y = t^2 - 8t,\ t > 0$

Show that the only stationary point on the curve has coordinates $(2\ln 2,\ -16)$

WORKING AT A*/A

(1) A curve has parametric equations
$x = 3\cos 4t - 4,\ \ y = \sin 2t + 3,\ \ 0 < t < \frac{\pi}{6}$

(a) Show that $\frac{dy}{dx} = k \operatorname{cosec} 2t$ where k is a constant to be found.

(b) Show that there is a point on the curve where the tangent has a gradient of $-\frac{1}{6}$.

(2) The curve C has parametric equations

$x = 2\cos\frac{t}{2},\ y = 1 - \sin 2t,\ \ 0 \le t \le \pi$

Point P on the curve C has coordinates $(\sqrt{2}, 1)$

(a) Find the value of t at the point P.

(b) Find the equation of the tangent to the curve at the point P in the form $ax + by = c$

(3) A curve has parametric equations
$x = 4t^2,\ y = t^2 - 8t,\ t \epsilon R$

(a) Find an equation for the tangent to the curve at the point where $t = 0.5$

(b) Prove that this is the only point the where the tangent meets the curve.