WWW.M4THS.COM
A LEVEL MATHS

(50) Differentiation using the Quotient Rule

WORKING AT D/E

(1) For each of the following, state which rule would be used to differentiate the function. Write Product (P), Quotient (Q), Chain (C) or None of these (N).

(a) $f(x) = \frac{x}{\sin x}$ (b) $f(x) = xe^x$ (c) $f(x) = \frac{\ln}{x}$

(d) $f(x) = (2x-4)^{-5}$ (e) $f(x) = e^{3x}$

(2) Using the formula book, or otherwise,

(a) show that if $y = \frac{x}{\ln x}, x > 1$ then $\frac{dy}{dx} = \frac{\ln x - 1}{[\ln x]^2}$

(b) Hence, show that there is a stationary point when $x = e$ and $y = e$

(3) Using the formula book, find an expression for $g'(x)$ for each giving your answer in its simplest form.

(a) $g(x) = \frac{x}{\cos x}$ (b) $g(x) = \frac{x^2}{e^{3x}}$ (c) $g(x) = \frac{\ln}{\text{si}}$

WORKING AT B/C

(1) A curve has the equation $y = \frac{2x-1}{e^x}$, $x \in R$.

(a) Find a fully simplified expression for $\frac{dy}{dx}$

(b) Hence, an equation for the normal to the curve at the point where $x = 1$

(2) $y = \frac{x^3+1}{\cos x}$, $0 \leq x < \frac{\pi}{2}$

Find $\frac{dy}{dx}$

(3) $f(\theta) = \frac{\theta}{\sin\theta}$, $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$

(a) Find $f'(\theta)$

(b) Hence, show the stationary point on the curve satisfies the equation $\theta = \tan\theta$

(c) Find $f'\left(\frac{\pi}{2}\right)$

(d) Hence, show that the equation of the tangent to the curve when $\theta = \frac{\pi}{2}$ is $y = \theta$

WORKING AT A*/A

(1) $y = \frac{x}{\sqrt{x-1}}$, $x > 1$

(a) Show that $\frac{dy}{dx} = \frac{x-2}{2(x-1)^{\frac{3}{2}}}$

(b) Find the equation of the tangent to the curve at the point where $x = 3$ giving your answer in the form $ax + by = c$ where a and c are integers and b is in exact form.

(2) Find the exact coordinates of any stationary point on the curve with equation $y = \frac{x^2}{e^{3x}}$, $x \epsilon R$

(3) $g(x) = \frac{2}{x} + \frac{3}{x^2+x}$, $x > 0$

(a) Show that $g(x) = \frac{2x+5}{x^2+x}$

(b) Prove that there are no stationary points on the curve of $y = g(x)$