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A LEVEL MATHS

(49) Differentiation using the Product Rule

WORKING AT D/E

(1) Circle all of the equations below where Cyril can use the product rule to find $\frac{dy}{dx}$.

(a) $y = x\cos x$ (b) $y = 3xe^{4x-2}$ (c) $y = x^2$

(d) $y = \sin(x)\sqrt{1-x^4}$ (e) $y = \ln(2x+7) + 3x$

(2) Using the formula book, or otherwise, find $\frac{dy}{dx}$ for each of the following using the product rule:

(a) $y = xe^x$ (b) $y = x^2\cos x$ (c) $y = \sqrt{x}\sin x$

(3) A curve has equation $y = 3x\cos 2x,\ \ x\epsilon R$

(a) Show that $\frac{dy}{dx} = 3\cos 2x - 6x\sin 2x$

(b) Hence, show that the gradient of the tangent to the curve at the point where $x = 0$ is 3.

WORKING AT B/C

(1) A curve has equation $y = x(x-1)^{\frac{1}{2}},\ x > 1$

(a) Show that $\frac{dy}{dx} = \frac{3x-2}{2(x-1)^{\frac{1}{2}}}$

(b) By considering the domain, explain why there are no stationary point on the curve.

(2) $y = 2 + e^x\sin x,\ \ \frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$

(a) Find the equation of the tangent to the curve at the origin.

(b) Show that the stationary point on the curve satisfies the equation $\tan x = -1$

(c) Hence, find the exact coordinates of the stationary point.

(3) $f(x) = x\ln x,\ x > 0$

(a) Find an expression for $f'(x)$

(b) Hence, show that $f'(e^4) = 5$

WORKING AT A*/A

(1) A curve has equation $y = (x)\left(\sqrt[3]{x^2+1}\right),\ x\epsilon R$.

(a) Find the gradient of the curve at the point where $x = \sqrt{7}$ giving your answer in exact form.

(b) Explain why there is only one real root to the equation.

(2) $g(x) = 3e^{2x}\cos 2x,\ 0 < x < \frac{\pi}{2}$

Find the set of values of x such that $g(x)$ is an increasing function.

(3) A curve has equation $y = e^{-x}\ln x,\ \ x > 0$

(a) Show that $\frac{dy}{dx} = e^{-x}\left(\frac{1}{x} - \ln x\right)$

(b) By drawing two different graphs, write down the number of stationary points on the curve with equation $y = e^{-x}\ln x$

(c) Find the equation of the normal to the curve at the point where $x = 1$ giving your answer in the form $ax + by = c$.