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A LEVEL MATHS

(48) Differentiation using the Chain Rule

WORKING AT D/E

(1) Use the chain rule to find an expression for $\frac{dy}{dx}$ for each of the following.
(a) $y = (x^3 + 4)^6$ (b) $y = \cos^5 x$ (c) $y = 4\sin 2x$
(d) $y = \cos 8x$ (e) $y = e^{x^2}$ (f) $y = \ln(x^2 + 3x)$

(2) $y = e^{3x},\ x\epsilon R$
(a) Find an expression for $\frac{dy}{dx}$
(b) Find the value of y when $x = 1$ giving your answer in exact form.
(c) Hence, show that the equation of the tangent to the curve at the point where $x = 1$ is
$y = 3e^3x - 2e^3$

(3) $y = \sin 3x + \cos 3x\ 0 \le x \le \frac{\pi}{2}$
(a) Find an expression for $\frac{dy}{dx}$
(b) Hence, show that any stationary points satisfy the equation $\tan 3x = 1$
(c) Show that the stationary points have x coordinates $x = \frac{\pi}{12}$ and $x = \frac{5\pi}{12}$

WORKING AT B/C

(1) (a) $f(x) = \ln(2x^2 + 1),\ x > -1$

Show that the only stationary point on the curve is (0,0)

(b) $g(x) = \sqrt{3x^3 - x}\ ,\ -\frac{1}{\sqrt{3}} \le x \le 0$

(i) Show that $g'(x) = \frac{9x^2-1}{2\sqrt{3x^3-x}}$

(ii) Hence, find the only stationary point on the curve. Give your answer in exact form.

(2) A curve has equation $x = y^2 - y$
(a) Find an expression for $\frac{dx}{dy}$
(b) Hence, find the value of $\frac{dy}{dx}$ when $y = 3$

(3) A curve has equation $y = e^{x^2},\ x \in R$.

(a) Show that the only stationary point on the curve has coordinates (0,1)

(b) Show that the equation of the tangent to the curve at the point with x coordinate 1 is
$y = 2ex - e$

WORKING AT A*/A

(1) (a) Given that $g(x) = \ln\cos^2 x\,, 0 \le x < \frac{\pi}{2}$ show that $g'(x) = k\tan x$, where k is a constant to be found.

(b) Hence, find the coordinates of the stationary point on the curve of $y = g(x)$.

(2) $y = e^{sin3x},\ 0 \le x \le \frac{\pi}{2}$

Find the exact coordinates of any stationary points on the curve.

(3) A curve has equation $y = \ln(x^2 + 6x)\,,\ x > -6$
(a) Show that the equation of the tangent to the curve at the point where $x = 1$ can be written as
$y = \frac{8}{7}x + \ln 7 - \frac{8}{7}$

(b) Show that $f(x) = \ln(x^2 + 6x)\,,\ x > -6$ is an increasing function for all values of x.

(c) Find the only root of the equation $f(x) = 0$ giving your answer in exact form.