

(46) Differentiating $\sin x$ and $\cos x$ Functions

WORKING AT D/E

(1) Using the formula book, find $\frac{dy}{dx}$ for each of the following:

(a) $y = \cos 2x$ (b) $y = \sin 4x$ (c) $y = 5 \cos 3x$
(d) $y = -6 \sin 8x$ (e) $y = \sin(-3x) + 2 \sin(4x)$

(2) $f(x) = 2\sin(x) - x$, $0 < x < \frac{\pi}{2}$

(a) Find an expression for $f'(x)$

The curve with equation $y = f(x)$ has a stationary point P .

(b) Show that the x coordinate of P is $\frac{\pi}{3}$

(c) Hence, show that the coordinates of P are $(\frac{\pi}{3}, \sqrt{3} - \frac{\pi}{3})$

(3) $g(x) = \cos 2x$, $0 \leq x \leq \pi$

(a) Doris wants to find $g'(30^\circ)$. Can Doris do this?

(b) Find an expression for $g'(x)$

(c) Show that the gradient of the $g(x)$ at the point Q where $x = \frac{\pi}{4}$ is -2 .

(d) Hence, show that the equation of the tangent at Q is $y = -2x + \frac{\pi}{2}$

WORKING AT B/C

(1) $f(x) = \sin x + \cos x$, $0 \leq x \leq \pi$

(a) Show that the x coordinate of the stationary point on the curve of $y = f(x)$ satisfies the equation $\tan x = 1$.

(b) Hence, find the exact coordinates of the stationary point on the curve with equation $y = f(x)$.

(2) A curve has equation $y = \sin 2x - \cos 4x$.

(a) Find the equation of the tangent to the curve at the point $(\frac{\pi}{2}, -1)$

(b) Show that the tangent to the curve at the point $(\frac{\pi}{4}, 2)$ is a horizontal line.

(3) Show that there are no stationary points on the curve with equation $y = \sin 2x - 3x$, $0 \leq x \leq 2\pi$

WORKING AT A*/A

(1) Find the equation of the normal to the curve $y = 4 \sin x \cos x$ at the point with x coordinate $\frac{\pi}{3}$ giving your answer in the form $ax + by = c$

(2) A curve has equation

$$y = e^{2x} + 4 \cos 6x, \quad 0 < x \leq \frac{\pi}{4}$$

Show that the x coordinate of the stationary point on the curve satisfies the equation

$$x = \ln \sqrt{k \sin 2x}$$

where k is an integer to be found.

(3) Prove, from first principles, that the derivative of $\sin x$ is $\cos x$