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A LEVEL MATHS

(42) Trigonometric Identities for Parametric Equations

WORKING AT D/E

(1) A circle has parametric equations:

$$x = \cos\theta - 1, \qquad y = \sin\theta + 4,$$

(a) Using the identity $\cos^2\theta + \sin^2\theta \equiv 1$ find a cartesian equation for the circle.

(b) Hence, sketch the circle showing where the curve meets the coordinate axes.

(2) A curve has parametric equations:

$$x = \sec t, \qquad y = 2\tan t, \qquad 0 < t < \frac{\pi}{2}$$

(a) Use a trigonometric identity to show the cartesian equation of the curve is $y^2 = 4(x^2 - 1)$

(b) Explain why the range is $y > 0$

(3) A circle has parametric equations:

$$x = 5\cos\theta - 2, \qquad y = 5\sin\theta + 3,$$

(a) Find a cartesian equation for the circle.

(b) Write down the length of the radius of the circle.

WORKING AT B/C

(1) A curve has parametric equations:

$$x = \cos\theta + 1, \qquad y = \sin 2\theta, \qquad 0 < \theta < \pi$$

(a) Show that the cartesian equation can be written as $y = 2(x-1)\sqrt{x(2-x)}$

(b) Find the domain and the range of the function.

(2) A curve has parametric equations:

$$x = \tan^2 t, \qquad y = \frac{3}{\sin^2 t}, \qquad 0 < t < \frac{\pi}{2}$$

(a) Find a cartesian equation for the curve in the form $y = f(x)$.

(b) Find the domain and the range of $f(x)$

(3) A curve has parametric equations:

$$x = \cos 2t, \qquad y = \sin t, \qquad 0 < t \leq 2\pi$$

Find a cartesian equation in the form $x = f(y)$

WORKING AT A*/A

(1) A curve has parametric equations:

$$x = 4\cos t, \qquad y = \sin\left(t - \frac{\pi}{3}\right), \qquad 0 < \theta < \pi$$

(a) Show that the cartesian equation can be written as $y = \frac{1}{16}\left[2\sqrt{16 - x^2} - \sqrt{3}x\right]$

(b) Find the domain and the range of the function.

(2) A curve has parametric equations:

$$x = \cos 2t, \qquad y = \cot 2t, \qquad 0 < t < \frac{\pi}{4}$$

(a) Find a cartesian equation for the curve in the form $y^2 = f(x)$.

(b) Find the domain and the range of $f(x)$

(3) A cartesian equation is given by

$$(x+7)^2 + (y-3)^2 = 16$$

Write down the parametric equations of the circle in the form $x = f(t)$ and $y = f(t)$ stating a suitable domain for t.