

(40) Applications of Trigonometric Functions

WORKING AT D/E

(1) A dot is moving backwards and forwards across the full length of a computer screen. The position (P cm) of the dot after (t) seconds relative to the centre of the screen can be modelled by the equation

$$P = 10\cos 20t^\circ, \quad t \geq 0.$$

- (a) Explain why the width of the computer screen is 20 cm.
- (b) Find out where the dot was when it first started moving.
- (c) Calculate the time when the dot was first 4 cm to the left of the centre. Give your answer to 3SF.

WORKING AT B/C

(1) Cyril is making a mini wave machine for a science project. He places the device in a tank with water. The tank is in the shape of a cuboid. He positions a small rubber duck in the water and studies the height of the duck over a period of time. The duck's height (H cm) relative to the central height of the tank after (t) seconds can be modelled by the equation

$$H = 8 \sin \left(4t + \frac{\pi}{2} \right) + 2, \quad t \geq 0$$

where all angles are measured in radians.

- (a) Show that the initial height of the duck is 6 cm above the centre of the tank.
- (b) Find the maximum possible height above the centre of the tank the duck reaches.
- (c) Find how long it takes the duck to first reach this height giving your answer to 3SF.
- (d) Given that the depth of the tank is 30 cm, explain why the duck never hits the bottom of the tank.
- (e) Find the first time that the duck is at the central height of the tank to 3SF.

WORKING AT A*/A

(1) (a) Express $5 \cos x + 2 \sin x$ in the form $R \sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, giving your answer for R in exact form and α to 1 decimal place.

A robot is programmed to follow a path that can be modelled by the equation

$$H = 5 \cos \left(\frac{\pi t}{12} \right) + 2 \sin \left(\frac{t\pi}{12} \right) + 4, \quad 0 \leq t \leq 10$$

where H is the height of the robot relative to a fixed point and t is the time in seconds after the robot starts the path.

- (b) Find the maximum height of the robot.
- (c) Find the first time the robot reaches this maximum height giving your answer to 3SF.
- (d) Find to 2 decimal places, the time when the robot has a height of 5 metres above the fixed point.
- (e) Find to 2 decimal places, the time when the robot has a height of 1 metre below the fixed point.