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A LEVEL MATHS

(38) $a\cos x \pm b\sin x$ as $R\cos(x \pm \alpha)$ $R\sin(x \pm \alpha)$

WORKING AT D/E

(1) (a) Use the formula book to find the expansion of $\sin(x+\alpha)$

(b) Hence, write down the expansion of $R\sin(x+\alpha)$

(c) Using your answers to part (a) and (b), show that $3\cos x + 4\sin x$ can be written as $5\sin(x + 36.9^o)$

(2) (a) Using the expansion for $\cos(A+B)$ in the formula book, show that

$$12\cos x + 5\sin x = 13\cos(x - 22.6^o)$$

(b) Hence, show that the solutions to the equation $12\cos x + 5\sin x = 6.5$ are $x = 82.6^0$ and $x = 322.6^0$ for $0 < x < 360^o$

(3) Write $8\cos x - 6\sin x$ in the form $R\cos(x+\alpha)$ where $R > 0$ and $0 < \alpha < 90^o$

WORKING AT B/C

(1) (a) Express $8\cos x + 6\sin x$ in the form $R\sin(x+\alpha)$ where $R > 0$ and $0 < \alpha < 90^o$

(b) Hence, solve the equations below:

(i) $8\cos x + 6\sin x = 10,\ \ 0 \le x \le 360^o$

(ii) $8\cos 2y + 6\sin 2y = 5,\ \ 0 \le y \le 360^o$

(2) (a) $2\cos x + 4\sin x$ in the form $R\cos(x-\alpha)$ where $R > 0$ and $0 < \alpha < 90^o$. Give your answer for R in exact form and α to 1 decimal place.

(b) $f(x) = 2\cos x + 4\sin x,\ x \in R$. Using your answer to part (a), find the maximum value of $f(x)$.

(c) Write down the coordinates of the first maximum point of $f(x),\ x > 0$.

(d) Explain why there are no solutions to the equation $2\cos x + 4\sin x = 4.5$ for any value of x

(3) (a) Express $3\cos x + 2\sin x$ in the form $R\cos(x-\alpha)$ where $R > 0$ and $0 < \alpha < 90^o$

(b) Hence sketch the graph of
$y = 8\cos x + 6\sin x,\ \ 0 \le x \le 360^o$
including the coordinates of any stationary points and points where the curve meets or crosses the coordinate axes.

WORKING AT A*/A

(1) $g(x) = 10 - \cos 5x + \sin 5x$

(a) Show that $g(x)$ can be written in the form

$g(x) = p\sin(5x - q) + r,\quad p > 0, 0 < q < \frac{\pi}{2}$

where p, q and r are constants.

(b) Find the maximum value of $g(x)$ in exact form.

(c) Given that $0 \le x \le \pi$, find the values of x that that maximise $g(x)$, giving exact values.

(d) Sketch the graph of $y = g(x),\ \ 0 \le x \le \pi$. On the graph show the coordinates of any maximum or minimum points and the coordinates of any points where the graph meets or crosses the coordinate axes. Any non-exact values are to be given to 3SF.

(2) (a) Express $7\cos x - \sin x$ in the form $R\cos(x+\alpha)$ where $R > 0$ and $0 < \alpha < 90^o$.

$f(x) = \frac{5}{7\cos x - \sin x}$

(b) Find the least value of $|f(x)|$ as a simplified surd

(c) Express $f(x)$ in the form $P\sec(x+\alpha)$

(d) Hence, sketch the graph of $y = f(x)$ $0 \le x \le 360^o$ including any asymptotes.

(3) (a) Solve the equation $2\cos x - \sin x = \frac{\sqrt{5}}{2}$, $0 < x < 2\pi$, giving your answers to 3SF in radians.

(b) Hence, solve $\sin 4y - 2\cos 4y = \frac{\sqrt{5}}{2}$, $-\pi < y < \pi$, giving your answers in exact form.