WWW.M4THS.COM
A LEVEL MATHS

(37) The Factor and Remainder Theorem

WORKING AT D/E

(1) $f(x) = x^3 - 2x^2 - 13x - 10$
(a) Using the factor theorem, show that $(x + 1)$ is a factor of $f(x)$.
(b) Using the factor theorem, show that $(x - 2)$ is not a factor of $f(x)$.
(c) Given that $f(5) = 0$ and $f(-2) = 0$, fully factorise $f(x)$

(2) $g(x) = 2x^3 + x^2 + px + 12$
(a) Given that $(x - 3)$ is a factor of $g(x)$, show that $p = -25$
(b) Using long division, fully factorise $g(x)$
(c) Using your answer to part (b), solve $g(x) = 0$

(3) $h(x) = 3x^3 + bx^2 + cx + d$ where b, c and d are constants.
Given that $h(3) = 11$ and $h\left(-\frac{1}{3}\right) = -4$
(a) What statement can be made about the expression $(x - 3)$?
(b) What statement can be made about the expression $(3x + 1)$?

WORKING AT B/C

(1) $f(x) = x^3 - 2x^2 - 5x + 6$
(a) Use the factor theorem to find a linear factor of $f(x)$ in the form $(x + a)$. You must show full workings.
(b) Use polynomial division to express $f(x)$ in the form $f(x) = (x + a)(x + b)(x + c)$
(c) Hence, solve the equation $f(x) = 0$
(d) Draw the graph of $y = f(x)$ showing where the curve crosses the coordinate axes.

(2) $g(x) = 4x^3 + px^2 + qx - 12$
Given that $(x + 2)$ and $(4x + 1)$ are factors of $g(x)$, show, using the factor theorem, that:
(a) $p = -15$ and $q = -52$
(b) Hence, fully factorise $g(x)$ showing full workings. Calculator methods are not accepted.

WORKING AT A*/A

(1) $f(x) = ax^3 + bx^2 + cx - 2$ where a, b and c are constants.

Use the following 3 facts to solve the equation $f(x) = 0$

$$f(1) = 0$$

$$f\left(-\frac{2}{3}\right) = 0$$

When $f(x)$ is divided by $(x - 2)$ the remainder is 40

You must show full workings.

(2) $g(x) = x^4 + x^3 - 6x^2 + 6x - 72$
(a) Show that $g(3) = 0$
(b) Using your answer from part (a), express $g(x)$ in the form $g(x) = (Ax^3 + Bx^2 + Cx + D)(x + E)$
(c) Given further that $(x + 4)$ is the only other factor of $g(x)$, sketch the graph of $y = g(x)$ showing any points where the curve crosses the coordinate axes.