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A LEVEL MATHS

(32) Reciprocal Trigonometric Identities

WORKING AT D/E

(1) Using the identity $\sin^2\theta + \cos^2\theta \equiv 1$, show that:

(a) $\tan^2\theta + 1 \equiv \sec^2\theta$

(b) $1 + \cot^2\theta \equiv \text{cosec}^2\theta$

(2) Given that $\sec x = 4$, show that $\tan x = \pm\sqrt{15}$

(3) Using the identities in question (1), prove each of the following identities:

(a) $(\cot x + 1)^2 - \text{cosec}^2 x \equiv 2\cot x$

(b) $\tan^2\theta - \cot^2\theta \equiv \sec^2\theta - \text{cosec}^2\theta$

WORKING AT B/C

(1) (a) Show that the equation

$$3\cot^2\theta - 5\,\text{cosec}\,\theta + 1 = 0$$

can be written as

$$(3\,\text{cosec}\,\theta + 1)(\text{cosec}\,\theta - 2) = 0$$

(b) Hence, solve the equation

$$3\cot^2\theta - 5\,\text{cosec}\,\theta + 1 = 0, \quad 0 < \theta < 360$$

(2) Given that $\cot A = \frac{5}{12}$, $0 < A < 90$, find the value of:

(a) $\tan A$

(b) $\text{cosec}\,A$

(c) If $90 \le A < 180$ instead, how would this change the answers to part (a) and (b) in the question?

(3) Solve the equation $3\sec x = 4\,\text{cosec}\,x$, $0 \le x \le 2\pi$, giving your answers in radians to 3SF.

WORKING AT A*/A

(1) Find the exact solutions to the equation

$$\sqrt{3}\sec^2\theta - 2\tan\theta - 2\sqrt{3} = 0, \quad 0 \le \theta \le 2\pi,$$

(2) Prove the identity

$$\frac{\sec^4\theta - \tan^4\theta}{\tan^2\theta} \equiv \sec^2\theta\cot^2\theta + 1$$

(3) Given that:

$$A = p\,\text{cosec}\,x$$

$$B = q\cot x$$

(a) Show that $(Aq)^2 - (Bp)^2 = (pq)^2$, where p and q are non-zero constants.

(b) Given that x is an acute angle, show that $\cos x \equiv \frac{\sqrt{A^2-p^2}}{A}$

(c) Given further that $A = 2$ and $p = 1$, find the value of x giving your exact answer in radians.