

(21) Binomial Expansion of the form $(1 + x)^n$

WORKING AT D/E

(1) Using the formula book, find the first 4 terms in each of the expansions below in ascending powers of x , simplifying each term.

- (a) $(1 + 2x)^{\frac{1}{4}}$, $|x| < \frac{1}{2}$
 (b) $(1 - x)^{-1}$, $|x| < 1$
 (c) $(1 + \frac{x}{3})^{-3}$, $|x| < 3$

(2) $f(x) = \sqrt{1 + 4x}$, $|x| < \frac{1}{4}$

- (a) Show that $f(x)$ can be written in the form $(1 + 4x)^n$, where n is a rational fraction.
 (b) Hence, find the first 4 terms in the expansion of $f(x)$ in ascending powers of x , simplifying each term.
 (c) Find the simplified value for $f(-0.1)$
 (d) Using your answer to part (b), and a suitable value of x find an approximation for $\frac{\sqrt{15}}{5}$

(3) Show that the first 4 terms in the expansion of $\frac{1}{1+x}$, $|x| < 1$ in ascending powers of x are:

$$1 - x + x^2 - x^3$$

WORKING AT B/C

(1) Using the formula book, or otherwise, find the first 4 terms in each of the expansions below in ascending powers of x , simplifying each term. State the set of values for which each expansion is valid.

- (a) $\frac{1}{\sqrt{1-x}}$
 (b) $\sqrt[3]{1 + 0.25x}$

(2) Show that the first 3 terms in the expansion of

$$\frac{2+x}{(1-x)^2}, |x| < 1 \text{ in ascending powers of } x \text{ are}$$

$$2 + 5x + 8x^2 \dots \dots$$

- (3) (a) Find the 4 terms in ascending powers of x of the expansion of $\sqrt{1 - 2x}$ stating the set of values of x for which the expansion is valid.
 (b) By substituting $x = 0.4$ into your expansion, find an approximation of $\sqrt{5}$
 (c) By considering your answer to part (a), explain how you can find a more accurate approximation to $\sqrt{5}$

WORKING AT A*/A

(1) In the expansion of $(1 + px)^n$, $|x| < \frac{1}{p}$ the first 3 terms in ascending powers of x are $1 - 4x - 4x^2 \dots$

(a) Showing full workings, find the value of the rational constants p and n .

(b) Find the 4th term in the expansion.

(c) By choosing a suitable value of x use the expansion to find a cubic approximation to $\sqrt[3]{\left(\frac{47}{50}\right)^2}$

$$(2) g(x) = \frac{\sqrt{1+3x}}{1-2x}$$

Find the first 3 terms in ascending powers of x in the series expansion of $g(x)$ stating the set of values of x for which the expansion is valid.

(3) In the expansion of $\frac{2}{\sqrt[4]{1+ax}}$, $a < 0$ the coefficient of the term in x^2 is 1.25. Find the coefficient of the term in x^3 .