**Further Maths (Topics 1-4) Test 2 – www.m4ths.com – Steve Blades ©**

(1) The complex number $z$ satifies $|z-1-i| = |z-5+3i|$.
Find the least value of $|z|$ for points on this locus on the Argand diagram.

(2) The complex numbers $z_1 = 1 + i\sqrt{3}$ and $z_2 = a + bi$ are such that $\arg(z_1 z_2) = \frac{\pi}{6}$ and $\left|\frac{z_1}{z_2}\right| = 1$.
(a) Find $z_2$.
(b) Plot $z_2$ on an Argand diagram.

(3) The cubic equation $2x^3 - 4x^2 + 5x + 6 = 0$ has roots $\alpha, \beta$ and $\gamma$. Show that $\alpha^3 + \beta^3 + \gamma^3 = -16$

(4) Given that

$$\sum_{r=1}^{k} 4r = 840$$

Find the value of

$$\sum_{r=10}^{k} 4(r+1)(r-1)$$

(5) $z$ is a complex number. Given that $\frac{1}{z} = \frac{-(2+i)}{10}$, find $\arg(z)$ to 3SF.

(6) A quartic equation has roots $\alpha, \beta, \gamma$ and $\delta$.
Given that $\sum \alpha = 10$, $\sum \alpha\beta = -40$, $\sum \alpha\beta\gamma = 70$ and $\alpha\beta\gamma\delta = 39$ find an equation with integrer coefficients that has the roots $2\alpha, 2\beta, 2\gamma$ and $2\delta$.

(7) The complex number $z$ is represented by the point $P$ on the Argand diagram.
Given that $|z-4| = 2$ show that the maximum value of $\arg(z-1)$ in the interval $(-\pi, \pi)$ is $\arcsin\left(\frac{a}{b}\right)$ where $a$ and $b$ are integers in their simplest form.

(8) The equation $x^3 - x^2(p+8) + x(8p+41) - 41p = 0$ has roots $a, \beta$ and $\gamma$.
Given that $\sum \alpha\beta = 57$ find:
(a) The value of $\alpha\beta\gamma$
(b) The value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$